Exploring the Multidimensional Steady Flight Envelope of a Business Jet Aircraft

Silvia Ferrari

Princeton University

FAA/NASA Joint University Program on Air Transportation,

FAA Tech Center, Atlantic City, NJ

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Introduction

- Stringent operational requirements introduce:
  Complexity
  Nonlinearity
  Uncertainty

- Gain scheduling:
  Design a set of "local" linear controllers
  Interpolate local operating point designs

- Gain scheduling limitations:
  Nonlinearities dominate
  Fast transitions between operating points

- Nonlinear control design takes advantage of knowledge gained from linear controllers

Neural Networks for Control: Copying with Complexity

- Applicability to nonlinear systems
- Applicability to multi-variable systems
- Parallel distributed processing and hardware implementation
- Learning
- Flexible logic

Motivation for Neural Network-Based Controller

- Network of networks motivated by linear control structure

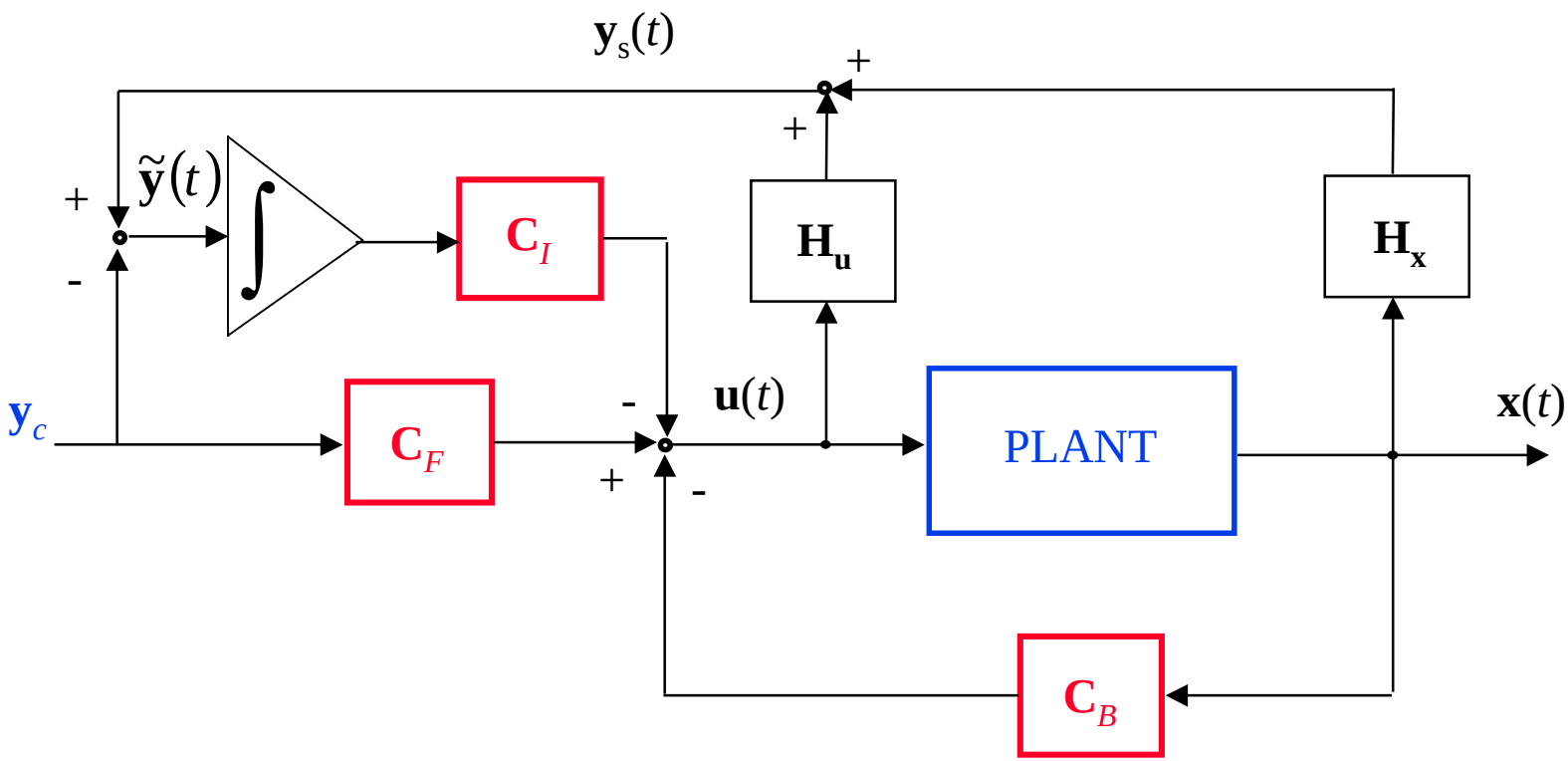
- Multiphase learning:
  Pre-training
  On-line training during piloted simulations or testing

- Improved global control

- Pre-training phase alone provides:
  A global neural network controller
  An excellent initialization point for on-line learning

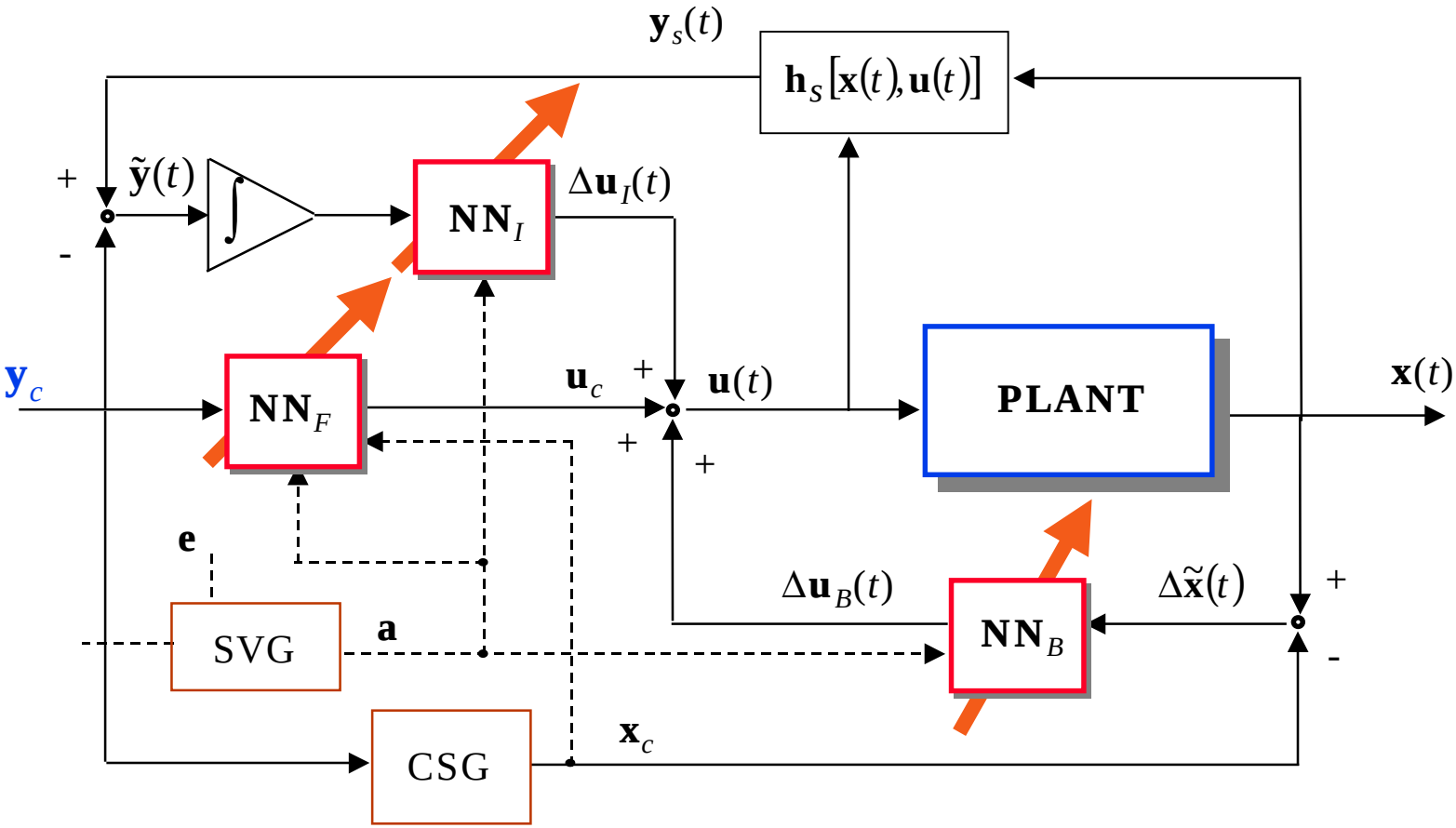
Proportional-Integral Controller

Closed-loop stability: $\mathbf{x}(t) \to \mathbf{x}_c, \quad \mathbf{u}(t) \to \mathbf{u}_c, \quad \tilde{\mathbf{y}}(t) \to 0$



$\tilde{\mathbf{y}}(t) = \mathbf{y}_s(t) - \mathbf{y}_c$, $\mathbf{y}_c$ = desired output, $(\mathbf{x}_c, \mathbf{u}_c)$ = set point.

Proportional-Integral Neural Network Controller



Where: $\mathbf{x}(t) \to \mathbf{x}_c, \quad \mathbf{u}(t) \to \mathbf{u}_c, \quad \tilde{\mathbf{y}}(t) \to 0, \quad \mathbf{y}_s(t) \to \mathbf{y}_c$

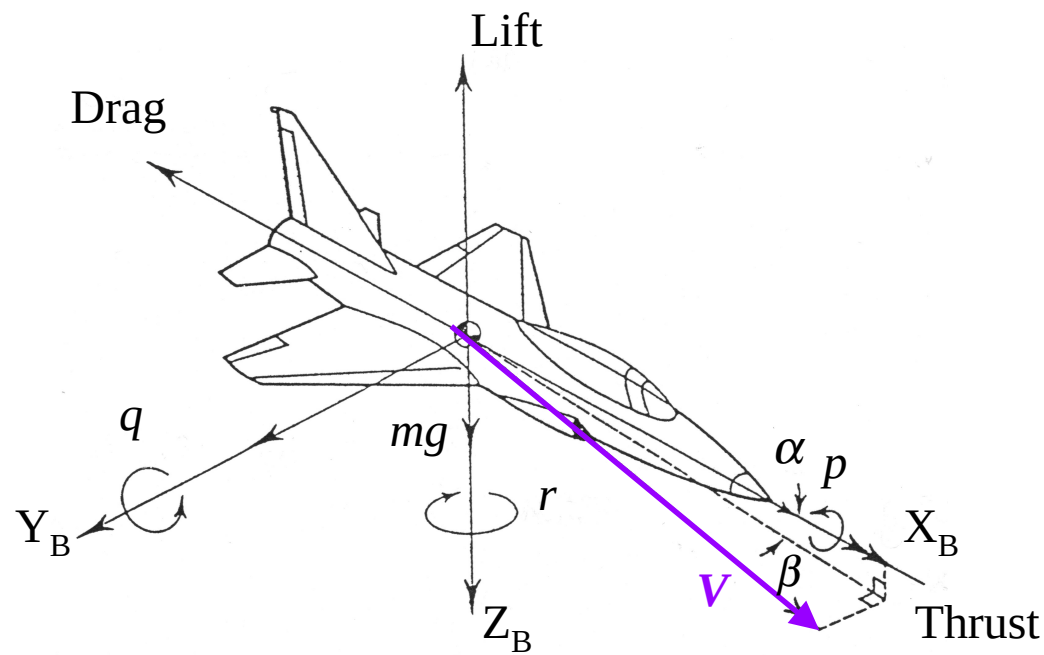
Nonlinear Business Jet Aircraft Model

Aircraft Equations of Motion:

$$\frac{d\mathbf{x}(t)}{dt} \equiv \dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t)]$$

$$\mathbf{y}(t) = \mathbf{h}[\mathbf{x}(t), \mathbf{p}(t), \mathbf{u}(t)]$$

Body axis: X_B, Y_B, Z_B



State vector: $\mathbf{x}(t) \in \Re^n$

Control vector: $\mathbf{u}(t) \in \Re^m$

Parameters: $\mathbf{p}(t) \in \Re^\ell$

Output vector: $\mathbf{y}(t) \in \Re^r$

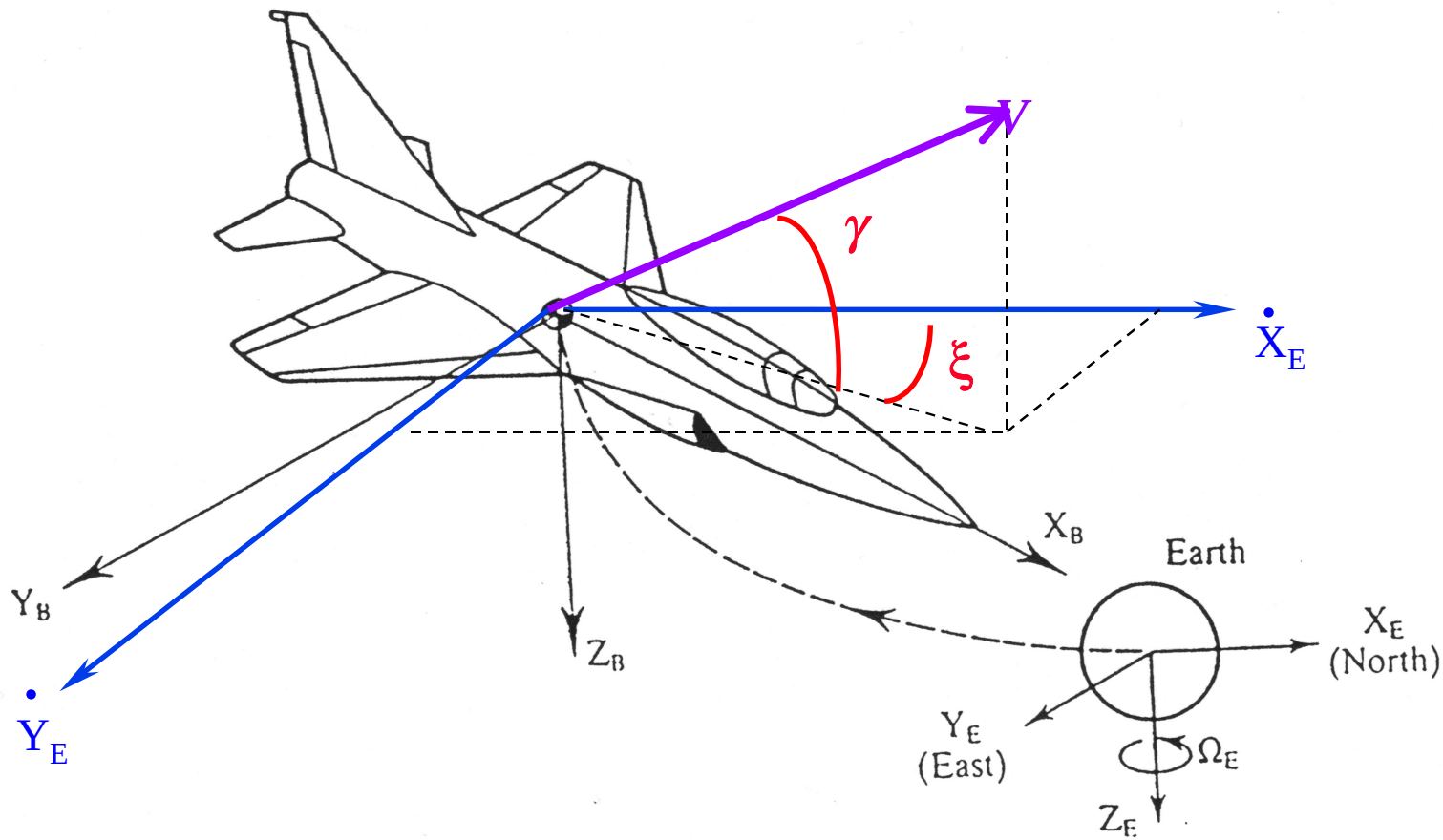
For the model:

$$\mathbf{x} \equiv [u \;\; v \;\; w \;\; x_e \;\; y_e \;\; z_e \;\; p \;\; q \;\; r \;\; \phi \;\; \theta \;\; \psi]^T$$

$$\mathbf{u} \equiv [\delta A \;\; \delta R \;\; \delta T \;\; \delta S]^T$$

Aircraft Body Frame and Flight Path Angles

Flight path angles: γ, ξ, μ

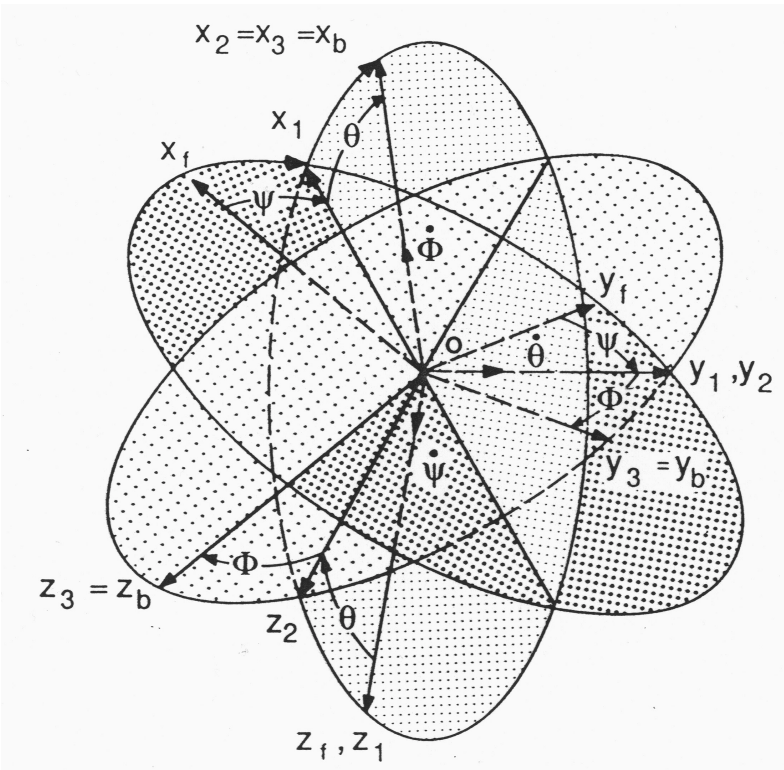


Command input: $\mathbf{y}_c = [V_c \;\; \gamma_c \;\; \mu_c \;\; \beta_c]^T$

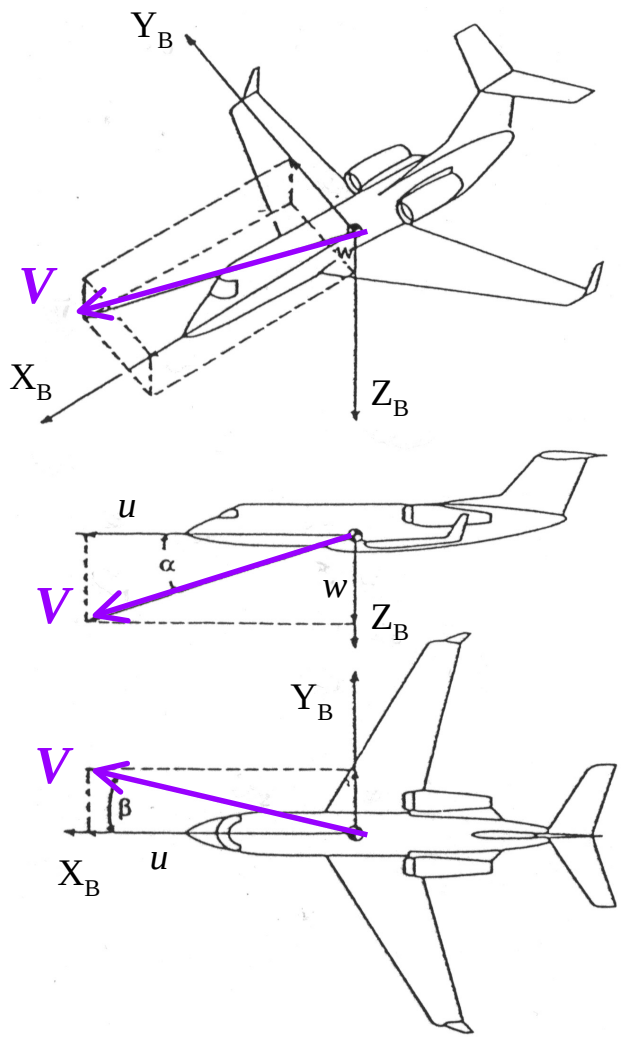
μ not shown on this figure

Aircraft Angles

Euler angles: ϕ, θ, ψ



Airflow angles: α, β



Kinematic equations:

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{L}_E^B \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

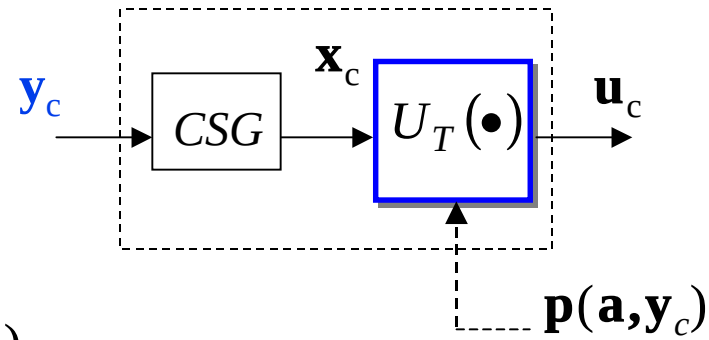
$\mathbf{L}_E^B$: non-orthogonal transformation matrix

Neural Network Modelling of Aircraft Trim Maps

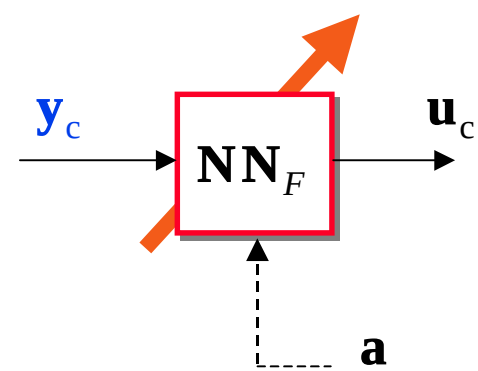
• The trim equations,

$$\mathbf{f}_T(\mathbf{x}_c, \mathbf{u}_c, \mathbf{p}) = 0,$$

are solved for the control settings, $\mathbf{u}_c$, over initial estimate of operational domain, (X_0, P_0).



• *Command State Generator (CSG)*: employs kinematic equations to produce $\mathbf{x}_c$



• The aircraft flight envelope consists of all $(\mathbf{x},\mathbf{p})$, for which $\exists$ a solution $\mathbf{u}_c$, s.t.:

$$U_T(\mathbf{x}_c, \mathbf{p}) \equiv \{\mathbf{u}_c : \mathbf{x}_c \in X_0,\ \mathbf{p} \in P_0,\ \textit{and}\ \mathbf{f}_T(\mathbf{x}_c, \mathbf{p}, \mathbf{u}_c) = 0\}$$

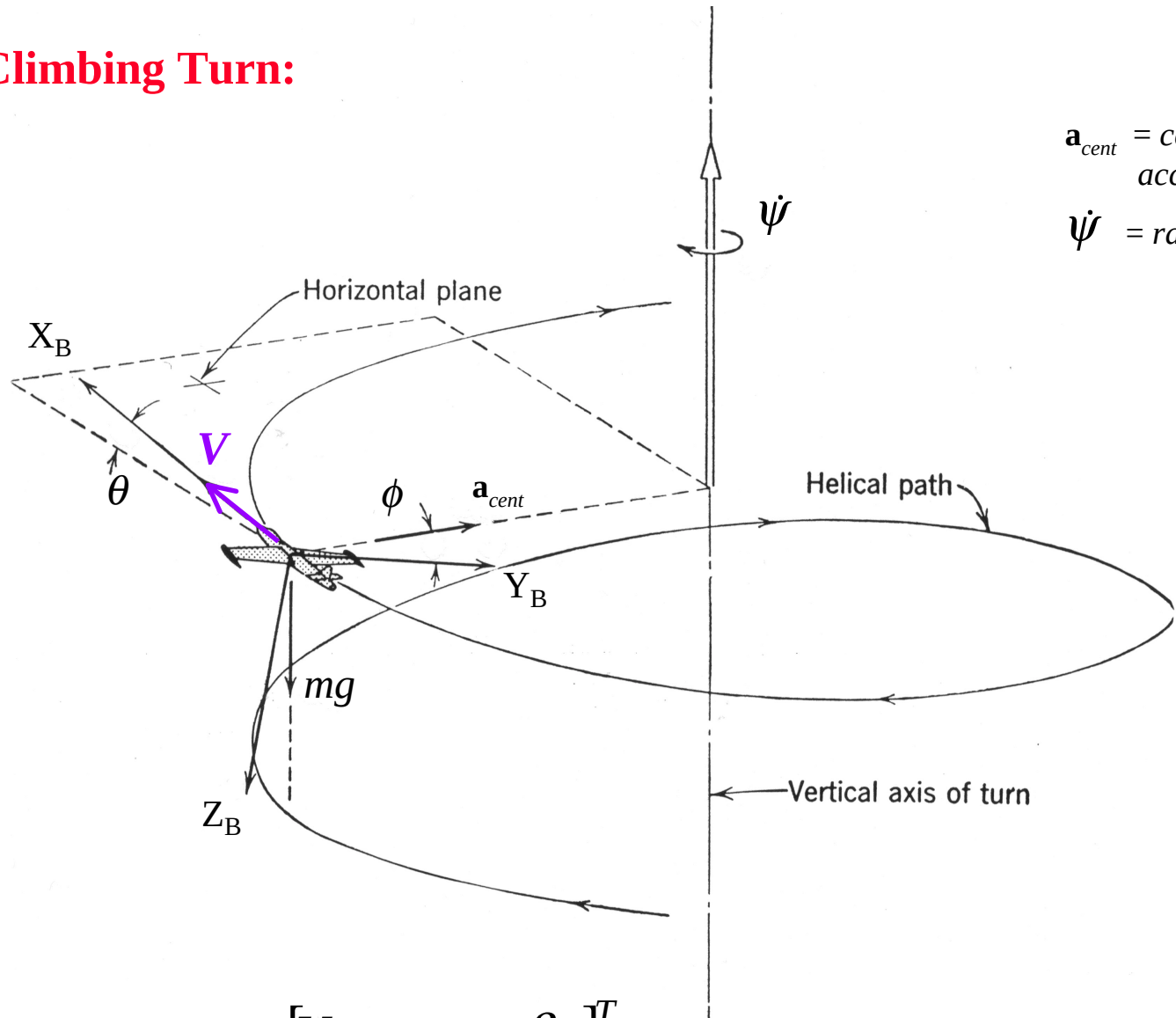
$\mathbf{a} = [\ h\ ]$

Example of Coordinated Maneuver

Steady Climbing Turn:

$\mathbf{a}_{cent}$ = *centripetal acceleration*

$\dot{\psi}$ = *rate of turn*

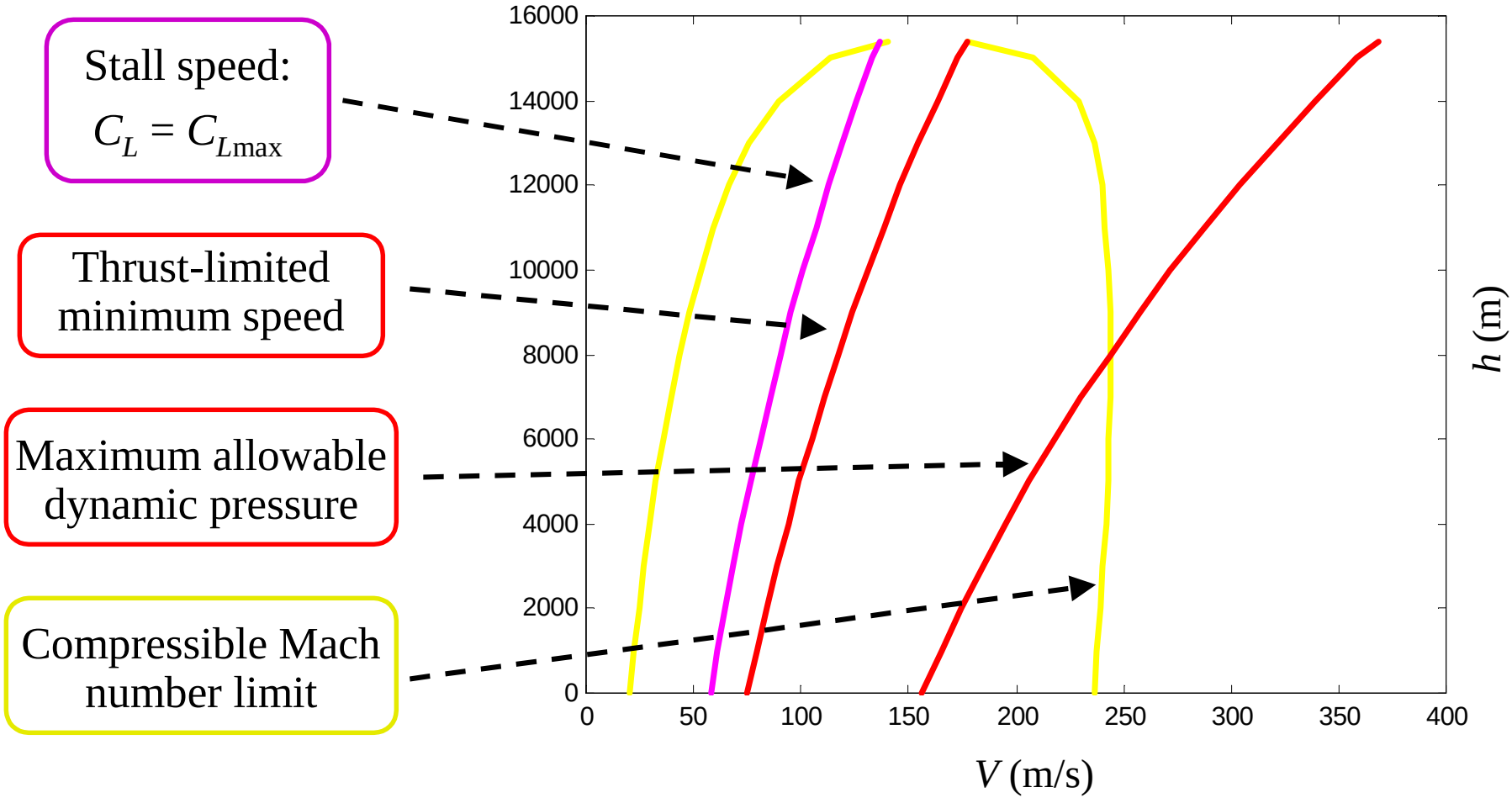


Command input: $\mathbf{y}_c = [V_c \;\; \gamma_c \;\; \mu_c \;\; \beta_c]^T$

[Etkin, "Dynamics of Flight"]

Estimated Business Jet Aircraft Steady Flight Envelope

Based on simplified aircraft model, and aerodynamics and thrust characteristics



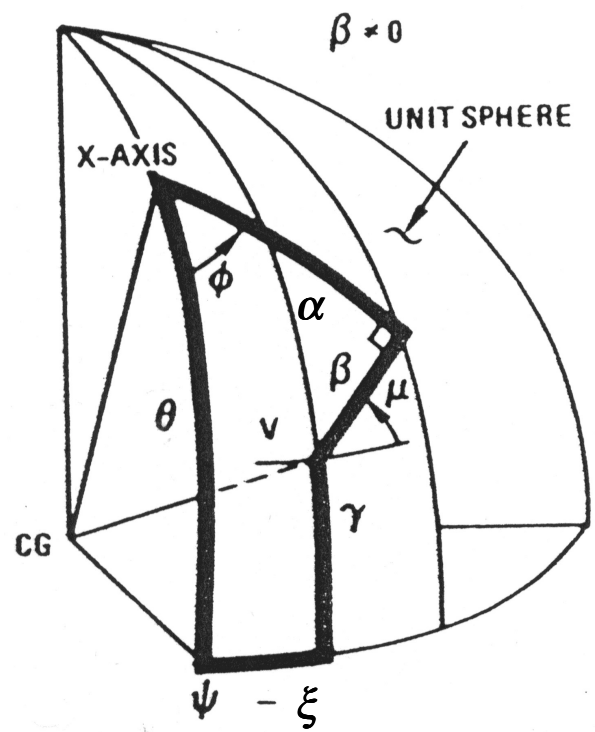
[Stengel, “Flight Dynamics and Control”]

Spherical Mapping of Aircraft Angles

- **Spherical trigonometric relations:**

$$(\gamma,\xi,\mu) = fn(\alpha,\beta,\phi,\theta,\psi)$$
$$(\phi,\theta,\psi) = fn(\alpha,\beta,\gamma,\xi,\mu)$$
$$(\alpha,\beta) = fn(\phi,\theta,\psi,\gamma,\xi)$$

- Angles defined on a unit sphere:



- **Redundant set of angles: ϕ, α**

$$\sin\phi = \frac{(\cos\gamma\sin\mu\cos\beta - \sin\gamma\sin\beta)}{\cos\theta},$$

$$\sin\alpha = \frac{(\cos\gamma\sin\theta\cos\phi - \sin\gamma\cos\theta\cos\phi)}{\cos\beta},$$

assuming $\xi = \psi$.

[Kalviste, 1987]

Non-Symmetrical Trim Solution at One Operating Condition

Given the command input, $\mathbf{y}_c = [V_c \;\; \gamma_c \;\; \mu_c \;\; \beta_c]^T$, and the scheduling variable, h,

In a coordinated maneuver: $\dot{\phi} = 0,\ \dot{\theta} = 0,\ \dot{\psi} = const.$

Solve the trim equations,

$$\mathbf{f}_T(\mathbf{y}_c, h) \equiv [\dot{u} \;\; \dot{v} \;\; \dot{w} \;\; \dot{p} \;\; \dot{q} \;\; \dot{r}]^T(\mathbf{y}_c, h) = 0 \text{, for } \dot{\psi}_c,\ \theta_c, \text{ and } \mathbf{u}_c = \begin{bmatrix} \delta A_c \\ \delta R_c \\ \delta T \\ \delta S_c \end{bmatrix}$$

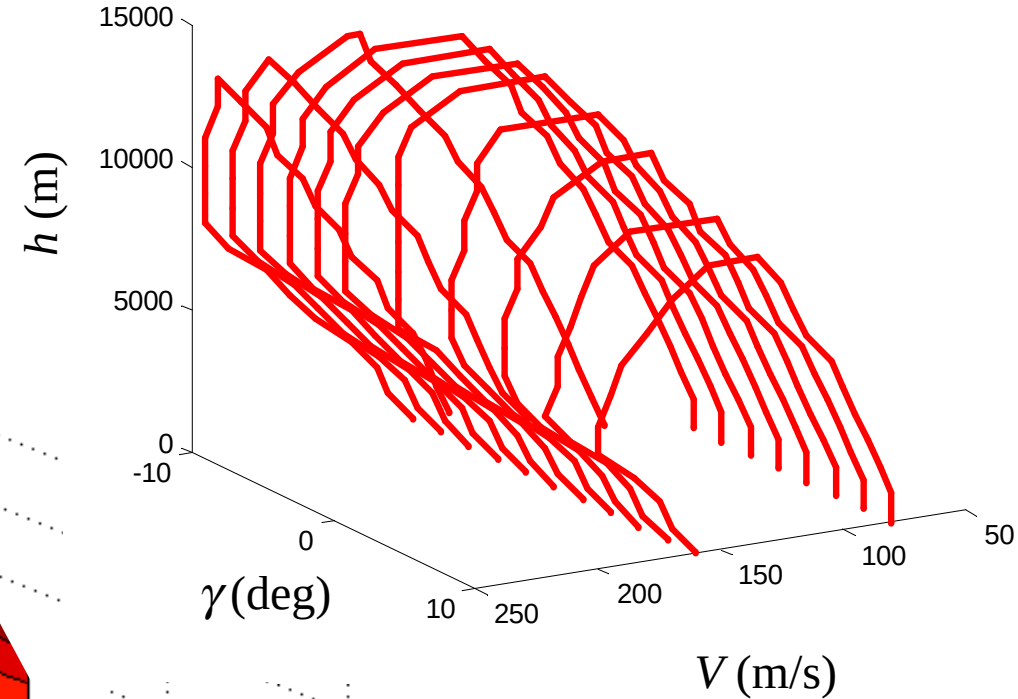
Subject to,

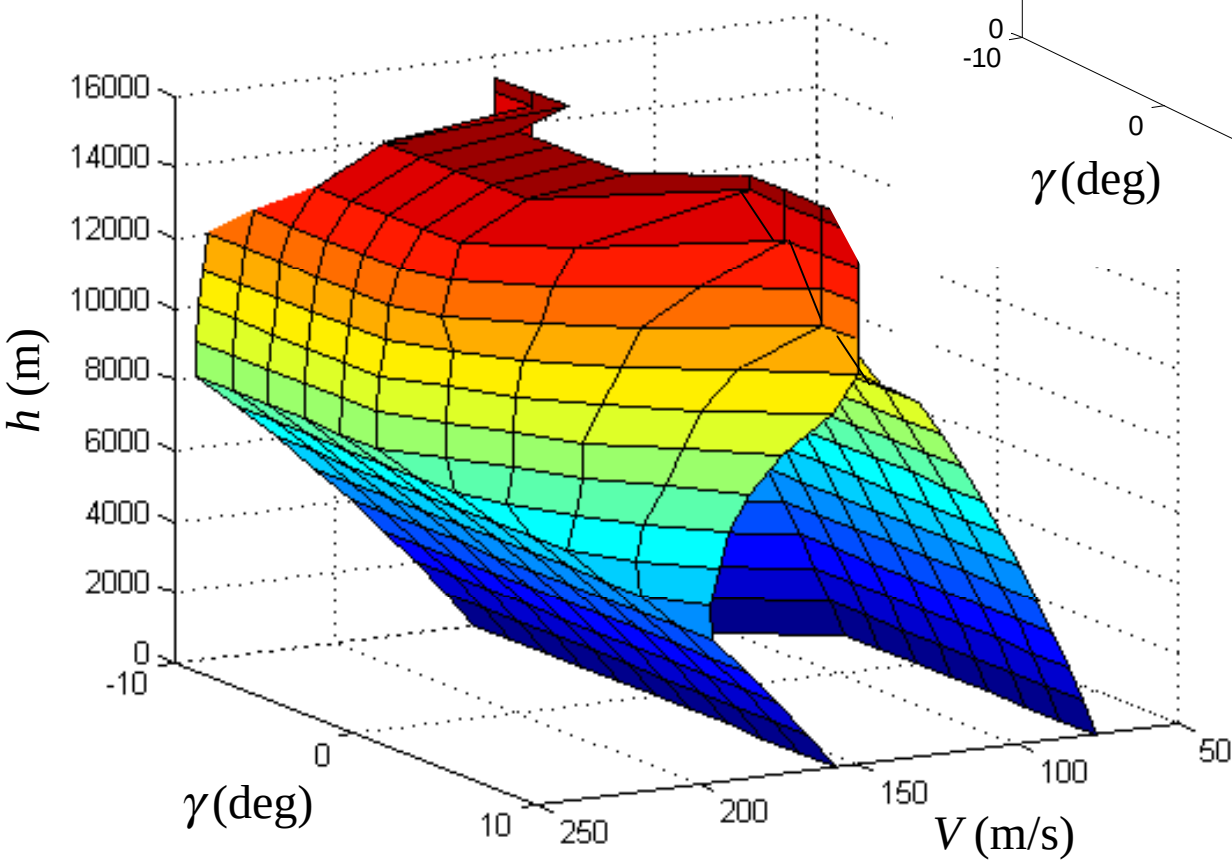
- Aircraft's kinematic equations: $$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} -\dot{\psi}\sin\theta \\ \dot{\psi}\cos\theta\sin\phi \\ \dot{\psi}\cos\theta\cos\phi \end{bmatrix}$$

- Spherical trigonometric relations: $$(\phi, \alpha) = fn(\theta, \gamma, \mu, \beta)$$

- Airflow angles definitions: $$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} V\cos\alpha\cos\beta \\ V\sin\beta \\ V\sin\alpha\cos\beta \end{bmatrix}$$

Computed Three-Dimensional Flight Envelope

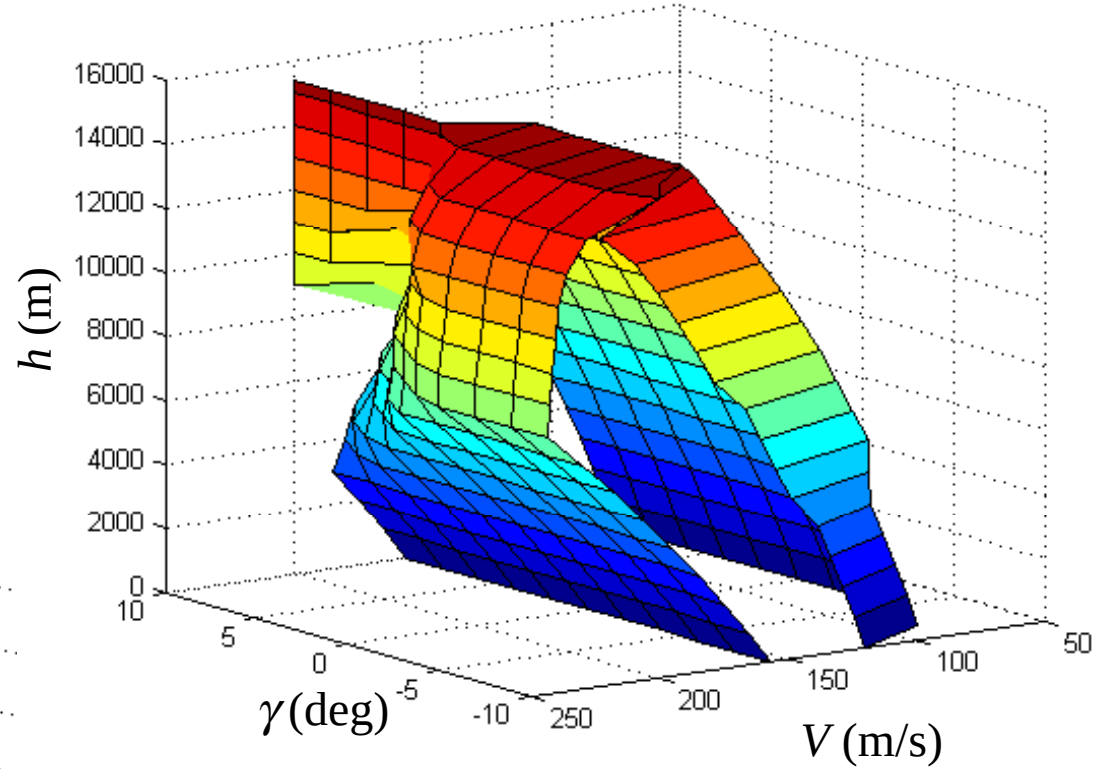


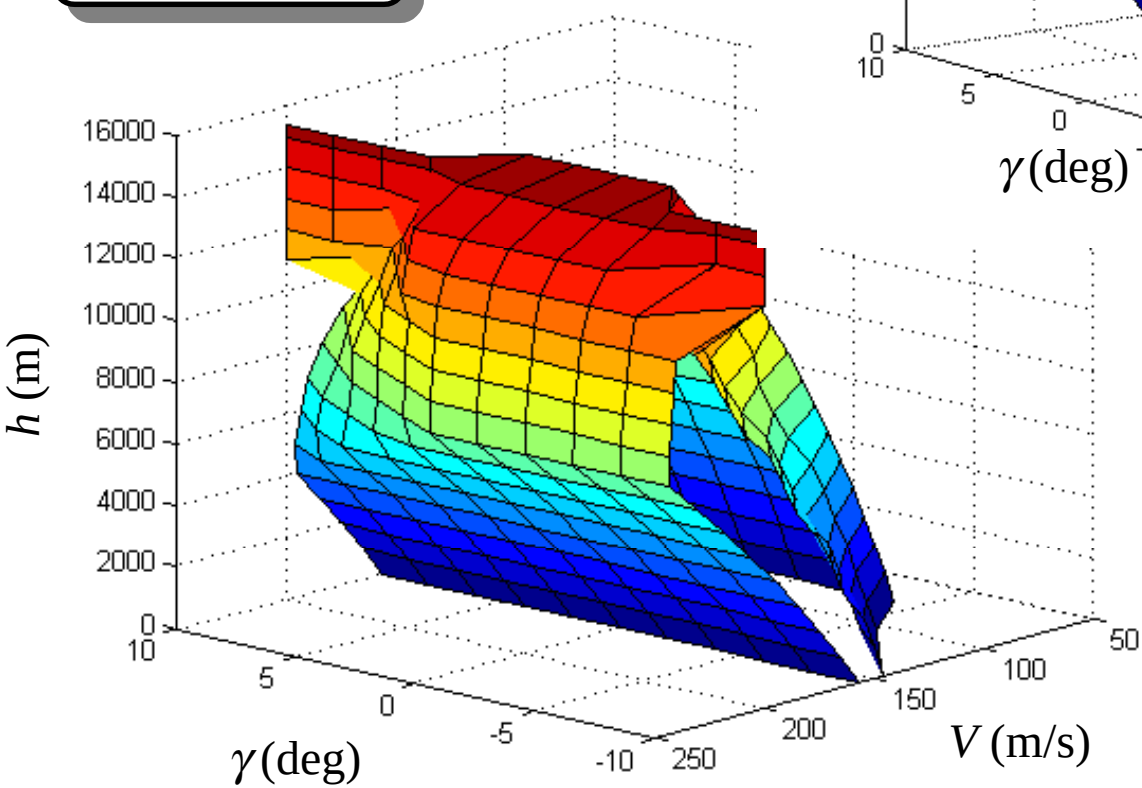


$$\beta = \mu = 0$$

Computed Multi-Dimensional Flight Envelope

$\beta = \mu = 0$





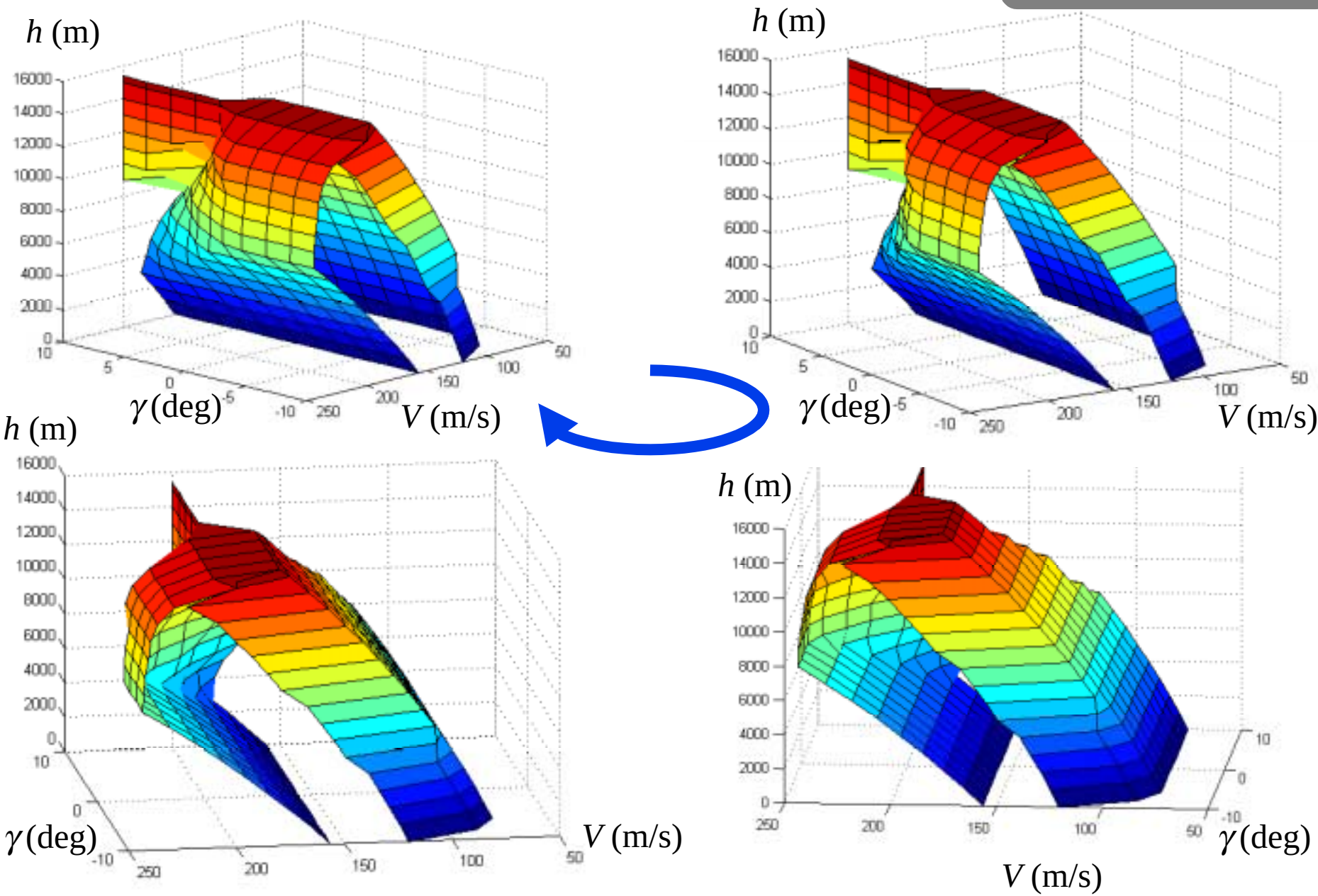
$\beta = 5, \mu = -20$

Similarly for all:

$$\begin{cases} \beta = \beta_{\min} : \beta_{\max} \\ \mu = \mu_{\min} : \mu_{\max} \end{cases}$$

Rotations of Three-Dimensional Envelope View

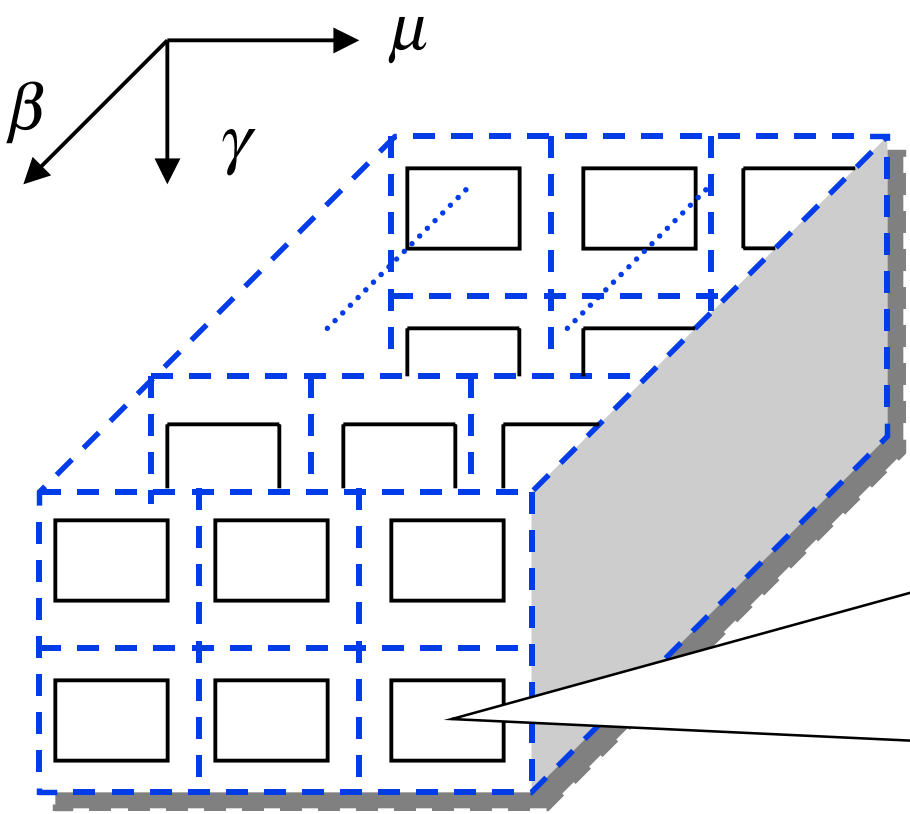
$\beta = 5, \mu = -20$



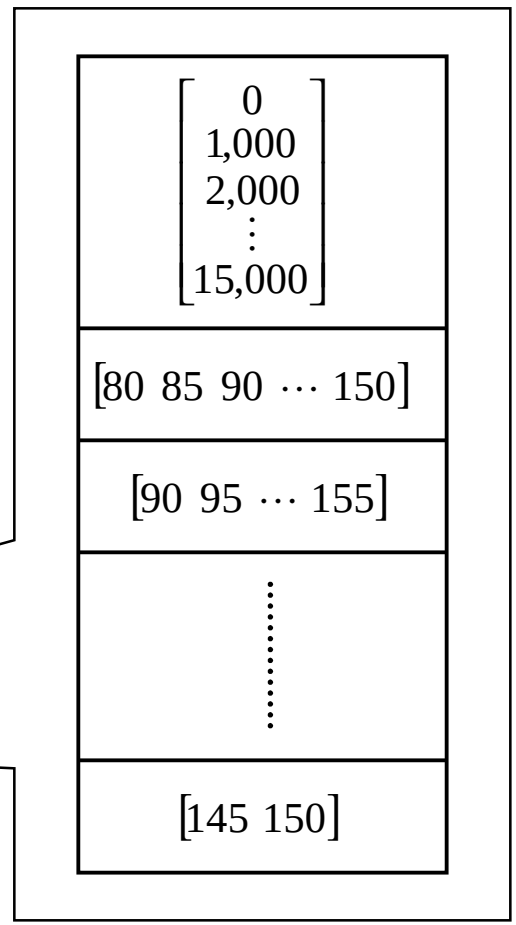
Multidimensional Flight Envelope Storage Using Cell Arrays

Cell arrays are containers that can hold other MATLAB arrays

Multidimensional envelope

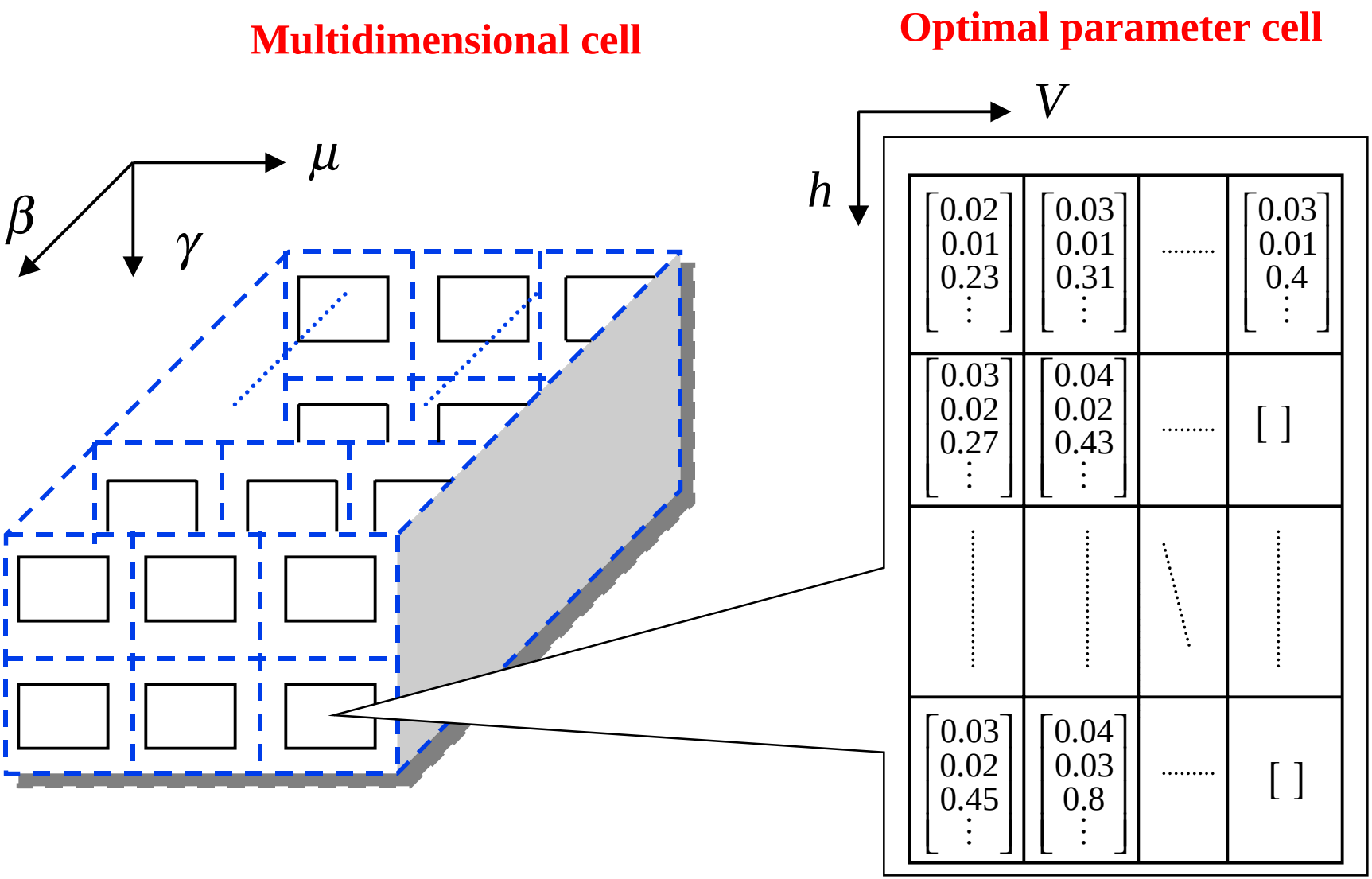


V-H envelope (nested cell)

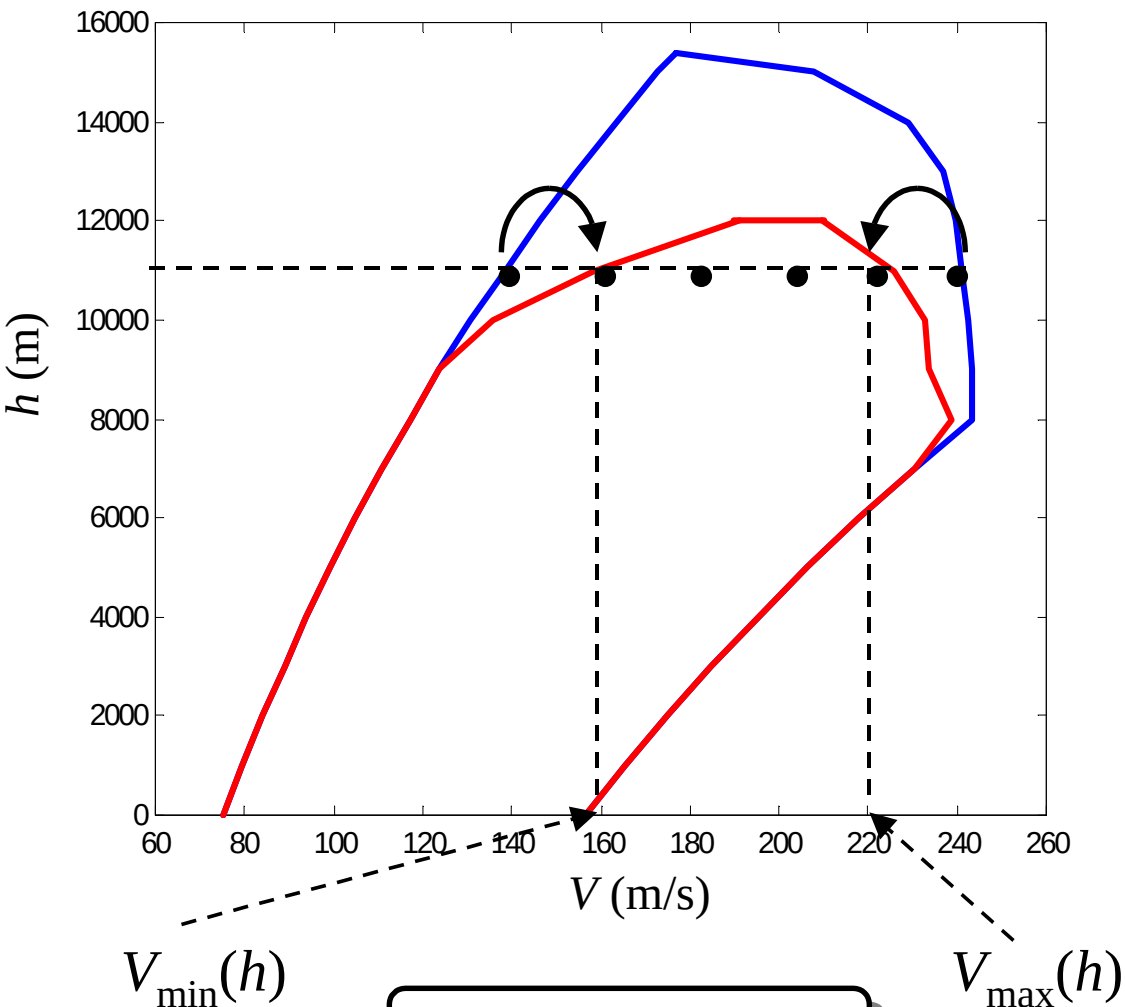


Optimal Parameters Storage Using Cell Arrays

To each operating point, there corresponds a vector of optimal parameters

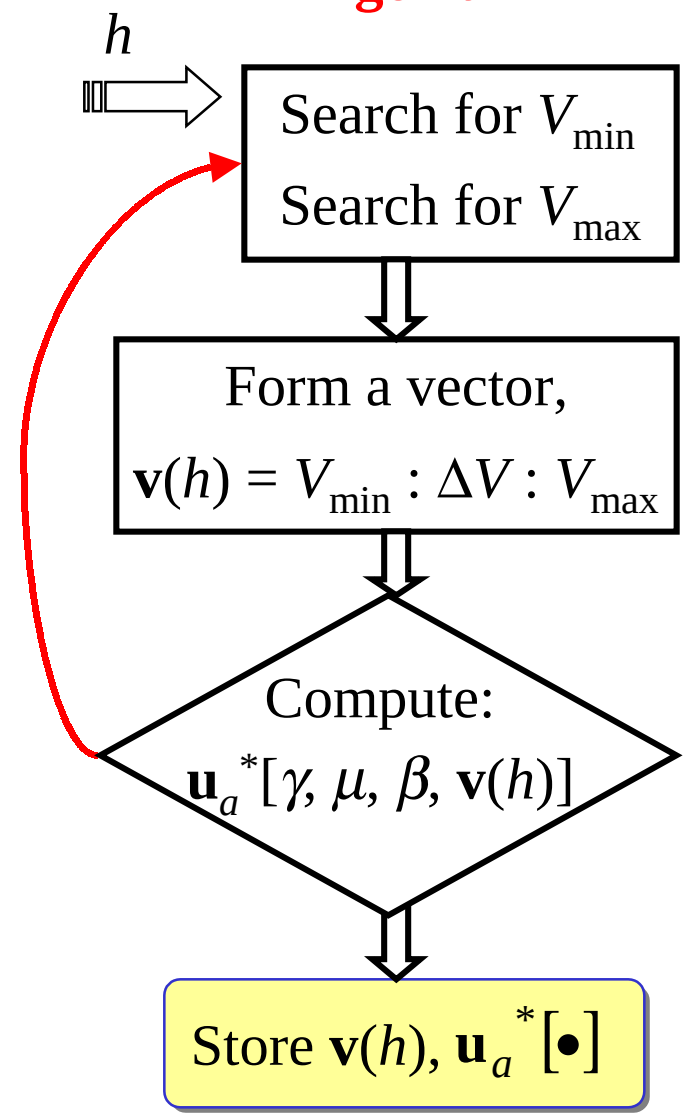


Computation of Trim Optimal Parameters Throughout Flight Envelope



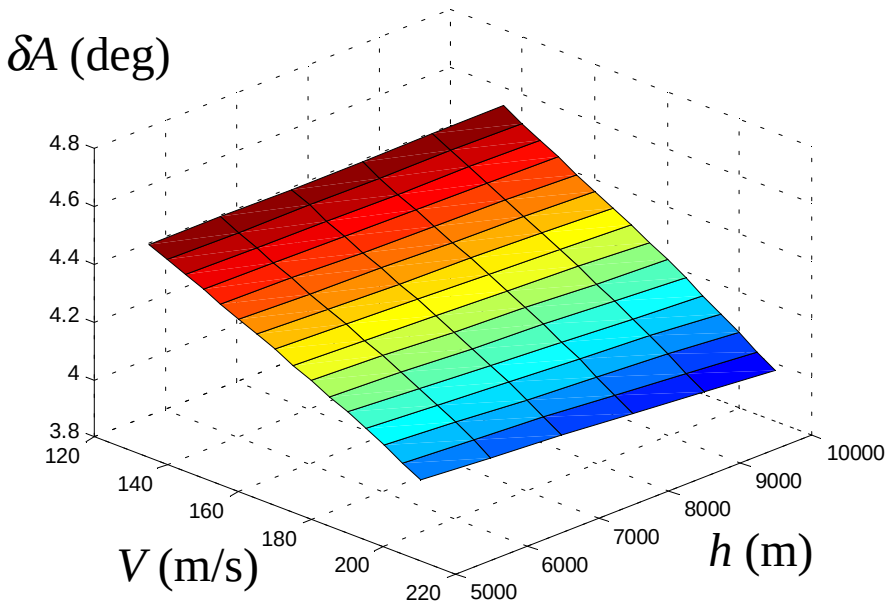
$\gamma = 0,\ \beta = 0,\ \mu = -50$

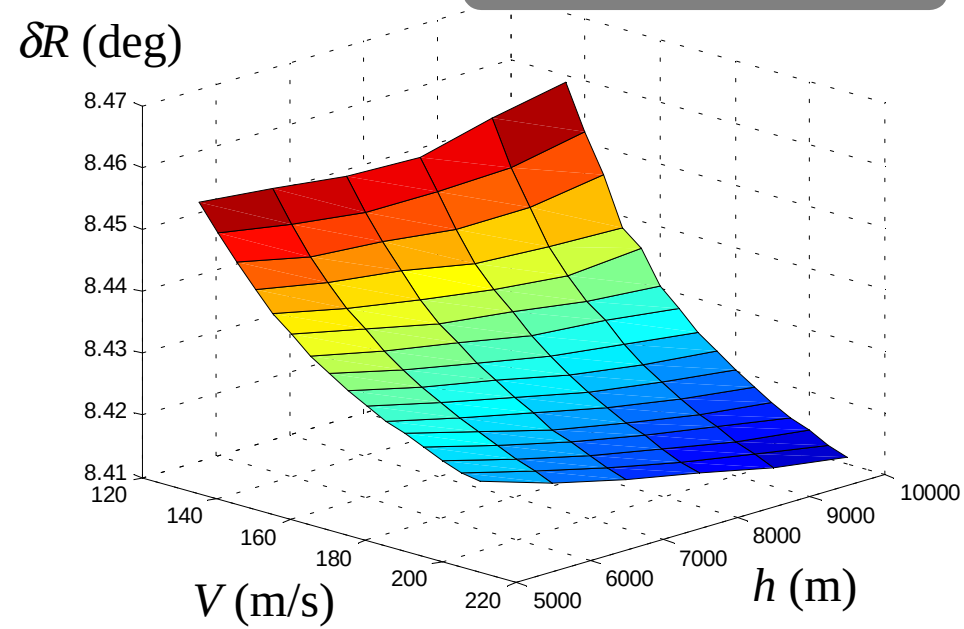
Algorithm

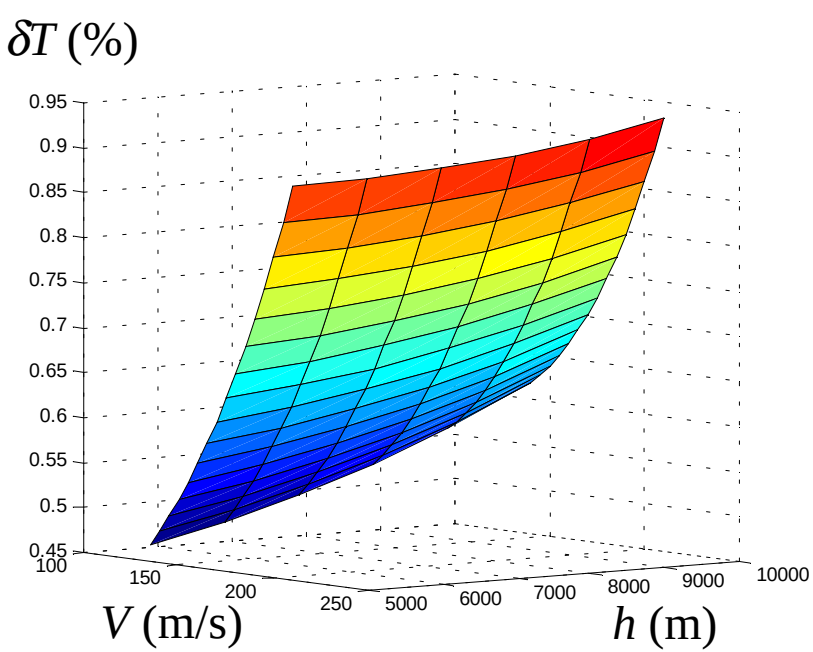


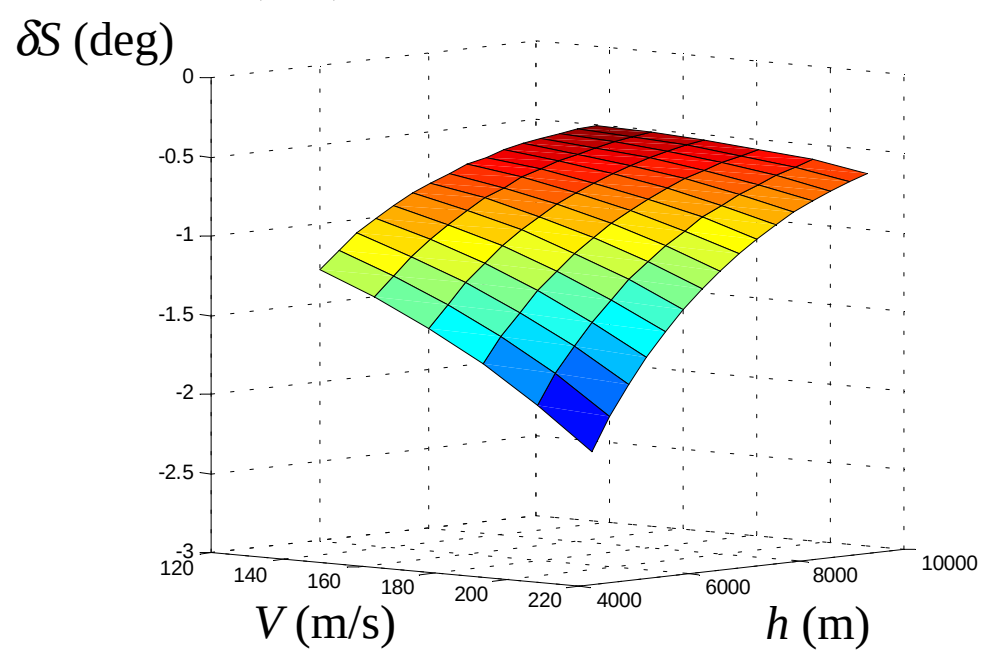
Example: Business Jet Aircraft Trim Surfaces

$\gamma = 3,\ \beta = 5,\ \mu = -10$









Summary and Conclusions

Non-symmetrical trim study:

- Full, nonlinear, coupled aircraft dynamics
- Trim solution at one operating point
- Multidimensional flight envelope computation
- Multidimensional envelope and trim data storage
- Results: trim control surfaces