Two-Dimensional Shape Optimization with Unstructured Meshes

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FAA/NASA Joint University Program for Air Transportation
Quarterly Review
MIT
October 18, 2002
Motivation for Automatic Design

• Aerodynamic development typically “cut&try”
  – Slow (design time doing detailed design iterations)
  – Expensive
  – Relies on physical insight of designer for changes

• Automatic design to reduce time in detail design phase
  – Improved performance
  – Decreased costs
Aerodynamic Shape Optimization

• Large number of design variables necessary for complete aircraft
• Control theory - gradient requires only the solution of an adjoint system
• Gradient calculation independent of the number of design variables
Design Procedure

1. Solve the flow equations for $r, u_1, u_2, u_3, p$
2. Solve the adjoint equations for $y$ subject to appropriate boundary conditions
3. Evaluate the gradient $G$
4. Project $G$ into an allowable subspace that satisfies any geometric constraints
5. Update the shape based on the direction of steepest descent
6. Return to step 1 until convergence is reached
Euler Flow Solver/Design Code SYN75

• 2D compressible inviscid fluid flow
• Finite volume
• Explicit multistage scheme of Jameson
• Multigrid time stepping scheme of Jameson
• Adjoint formulation for design problem
• General triangular mesh
• Equivalent to Galerkin finite element method
Governing Equations

Euler equations for flow of a compressible inviscid fluid in integral form:

\[
\frac{\partial}{\partial t} \iiint \omega d\Omega + \iint \mathbf{F} \cdot d\mathbf{S} = 0
\]

mass \quad momentum (for x) \quad energy
\[
\mathbf{w} = \rho \quad \mathbf{w} = \rho \mathbf{u} \quad \mathbf{w} = \rho E
\]
\[
\mathbf{F} = (\rho \mathbf{u}, \rho \mathbf{v}) \quad \mathbf{F} = (\rho \mathbf{u}^2 + p, \rho \mathbf{u} \mathbf{v}) \quad \mathbf{F} = (\rho \mathbf{H} \mathbf{u}, \rho \mathbf{H} \mathbf{v})
\]

\[
E = \frac{p}{(\gamma - 1)\rho} + \frac{1}{2}(u^2 + v^2)
\]

Equation of State
Governing Equations

Previous equations are solved for each CV in computational domain

\[
\frac{d}{dt} (V_i)w_i + \sum_k R_k = 0
\]

Flux contributions across interior faces cancel

\[
\frac{d}{dt} (V_i)w_i + \sum_k F_k \cdot S_k = 0
\]
Artificial Dissipation

Central differencing of the convective flux term allows oscillation of the solution due to even-odd decoupling.

Add dissipation terms to the Euler fluxes
Integration In Time
Multistage Scheme

Jameson’s Runge-Kutta method
for a m-stage scheme:

\[ w^{(0)} = w^n \]

\[ w^{(k)} = w^{(0)} - \alpha_k \Delta t \frac{1}{V} R(w)^{k-1} k = 1, 2, 3, \ldots m \]

\[ w^{(n+1)} = w^m \]
Convergence Acceleration

- Time step constraint for explicit schemes is too restrictive
- Convergence can be enhanced by
  - Local time stepping
  - Multigrid
Multigrid

- Coarse meshes are used to estimate the correction to the residual calculated on the fine mesh
- For minimal extra computational cost and memory, convergence rates are increased by an order of magnitude
- First utilized for Euler equations on an unstructured grid by A. Jameson
Multigrid: Fine Grid

Fine mesh 160x32 points
(view of partial mesh)

Trailing edge detail
Multigrid: Coarse Grids

80x16

40x8

20x4

10x2
SYN75 Results

RAE drag minimization

Initial solution

$C_D = 0.0062$
SYN75 Results

RAE drag minimization
15 design cycles
$C_D=0.0029$
Complete Configuration
Unstructured Mesh
Computational Requirements

• 2D - few design variables
  – Single processor

• 3D - large number of design variables
  – Serial computational time excessive
  – Parallel
    • Distribute work spatially: Domain Decomposition
Conclusion

- Adjoint formulation on unstructured mesh implemented
- Two-Dimensional unstructured mesh design tested
Work Plan

• Continue to develop 2D code
• Extend to 3D
  – Flow & adjoint solvers are in place (Jameson & Martinelli)
  – Write gradient formulation
• Shape modification
  – More general than point movement
  – Integration with CAD
• Pass Generals