Neural Network Control of a Hypersonic Inlet

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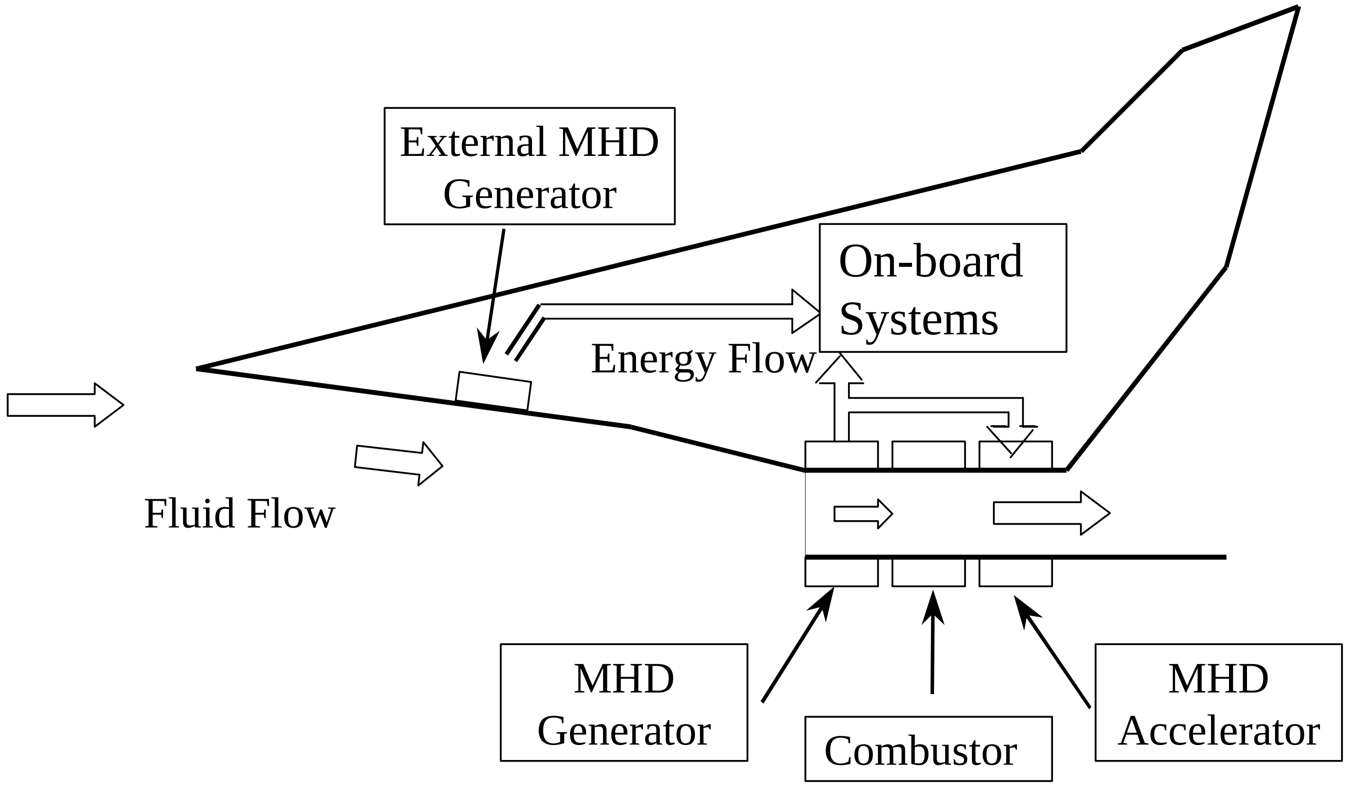
Presentation Outline

- The Magnetohydrodynamic (MHD) energy bypass engine
- Electron beam sustained MHD
- Analytical modeling
- The role of control
- Cost-to-go design for optimal control using Neural Networks
- Implementation details and preliminary results
- Summary and future work

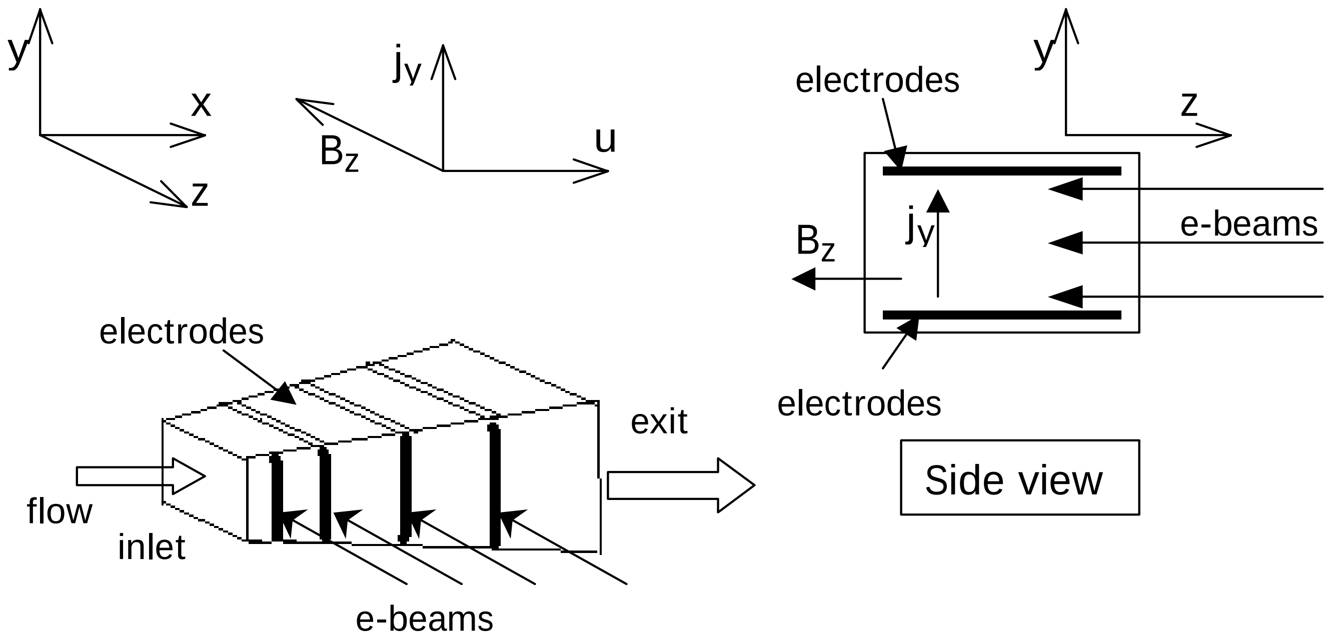
The MHD Energy Bypass Engine



Schematic of some of the technologies envisioned in the AJAX

1) Fraishtadt, V.L., Kuranov, A.L., and Sheikin, E.G., "Use of MHD Systems in Hypersonic Aircraft," Technical Physics, Vol. 43, No.11, 1998, p.1309.
2) Gurijanov, E.P., and Harsha, P. T., "AJAX: New Directions in Hypersonic Technology," AIAA Paper 96-4609, 1996.

Electron Beam Sustained MHD



Schematic of the MHD channel at the inlet

1) Macheret, S. O., Schneider, M. N., Miles, R. B., and Lipinski, R. J., "Electron Beam Generated Plasmas in Hypersonic Magnetohydrodynamic Channels," AIAA Journal, Vol. 39, No. 6, 2001, pp. 1127-1138.

Analytical Model

- Assumptions:
  - One-dimensional steady state flow
  - Inviscid flow
  - No reactive chemistry
  - Low Magnetic Reynolds number
- ‘*x*-*t*’ equivalence

Flow Equations

- Continuity Equation:

$$\frac{d(\rho u A)}{dx} = 0$$

x - coordinate along the channel

ρ - Fluid density

u - Fluid velocity

A - Channel cross-section area

- Force Equation:

$$\rho u \frac{du}{dx} + \frac{dP}{dx} = -(1-k)\sigma u B^2$$

P - Fluid pressure

k - Load factor

σ - Fluid conductivity

B - Magnetic field

Flow Equations…

- Energy Equation:

$$\rho u \frac{d(\gamma\varepsilon + \frac{u^2}{2})}{dx} = -k(1-k)\sigma u^2 B^2 + Q_\beta$$

ε- - Fluid internal energy

Q_β - Energy deposited by the e-beam

- Continuity Equation for the electron number density:

$$\frac{d(n_e u)}{dx} = \frac{2 j_b \varepsilon_b}{eY_i Z} - \beta n_e^2$$

n_e - Electron number density

j_b - Electron beam current

ε_b - E-beam energy

Z- Channel width

The Role of Control

- Electron beam current as the control element
- Maximizing energy extraction while minimizing energy spent on the e-beam ionization
- Minimizing adverse pressure gradients
- Attaining prescribed values of flow variables at the channel exit
- Minimizing the entropy rise in the channel

Performance Index

- Minimize

$$J = \begin{bmatrix} T(x_f) - T_e & M(x_f) - M_e \end{bmatrix} \begin{bmatrix} p_{11} & 0 \\ 0 & p_{22} \end{bmatrix} \begin{bmatrix} T(x_f) - T_e \\ M(x_f) - M \end{bmatrix} + p_{33} \left\{ C_v \ln\left[\frac{P(x_f)}{\rho(x_f)^{\gamma}} \right] - C_v \ln\left[\frac{P(0)}{\rho(0)^{\gamma}} \right] \right\}$$

$$+ \int_0^{x_f} \left[q_{11} Q_\beta A - q_{22} k(1-k) \sigma u^2 B^2 A + q_{33} h\left(\frac{dP}{dx}\right) \right] dx$$

Optimal Control using the Approach of Parametric Optimization

- For a given system:

$$x(k+1) = f[x(k), u(k), k]$$

- Parameterize a control law as:

$$u(k) = u[x(k), G]$$

- To maximize a performance index (minimize a cost function)

$$J = \phi[x(T), u(T)] + \sum_{i=1}^{T} \{L[x(i+1), u(i)]\}$$

- Equivalently minimize the cost-to-go function,

$$V(k) = \phi[x(T), u(T)] + \sum_{i=1}^{T} \{L[x(k+i, u(k+i-1)]\}$$

Motivating the cost-to-go approach

Linear time invariant system:

$$x(k+1) = Ax(k) + Bu(k)$$

Parameterizing,

$$u(k) = Gx(k)$$

$$\Downarrow$$

$$x(k+i) = (A+BG)^i x(k)$$

$$\Downarrow$$

$$V(k,G) = \frac{1}{2}\sum_{i=1}^{r}[x(k+i)^T Qx(k+i) + u(k+i-1)^T Ru(k+i-1)]$$

$$= \frac{1}{2}x(k)^T \sum_{i=1}^{r}[(A+BG)^{iT} Q(A+BG)^i + (A+BG)^{i-1T} G^T RG(A+BG)^{i-1}]x(k)$$

Modified Approach:

Parameterize as,

$$u(k) = G_1 x(k)$$

$$u(k+1) = G_2 x(k)$$

$$\ldots$$

$$u(k+r-1) = G_r x(k)$$

$$\Downarrow$$

$$x(k+i) = (A^i + A^{i-1}BG_1 + \ldots + ABG_{i-1} + BG_i)x(k)$$

$$\Downarrow$$

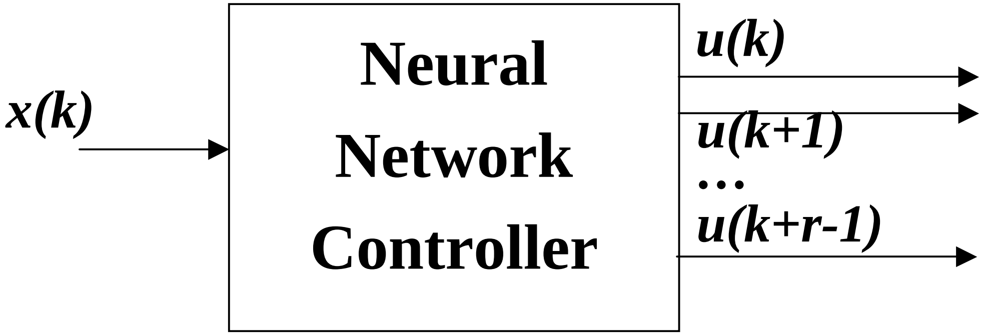
$$V(k) = \frac{1}{2} x(k)^T [(A^r + \ldots + ABG_{r-1} + BG_r)^T Q (A^r + \ldots + ABG_{r-1} + BG_r)$$

$$+ \ldots + (A + BG_1)^T Q (A + BG_1) + G_r^{\,T} R G_r + \ldots + G_1^{\,T} R G_1] x(k)$$

Solution with a unique minimum

Formulation of the Control Architecture
NN Controller

- Use of the modified approach to formulate the control architecture
- Instead of a single controller structure *(G)*, need *'r'* controller structures
- Outputs of the *'r'* controller structures, generate *u(k)* through *u(k+r-1)*
- Parameterize the *'r'* controller structures using an effective Neural Network



Formulation of the Control Architecture: NN Cost-to-go function Approximator

- Parameterize the cost-to-go function using a Neural Network (*CGA* Neural Network)
- Inputs to the *CGA* Network:

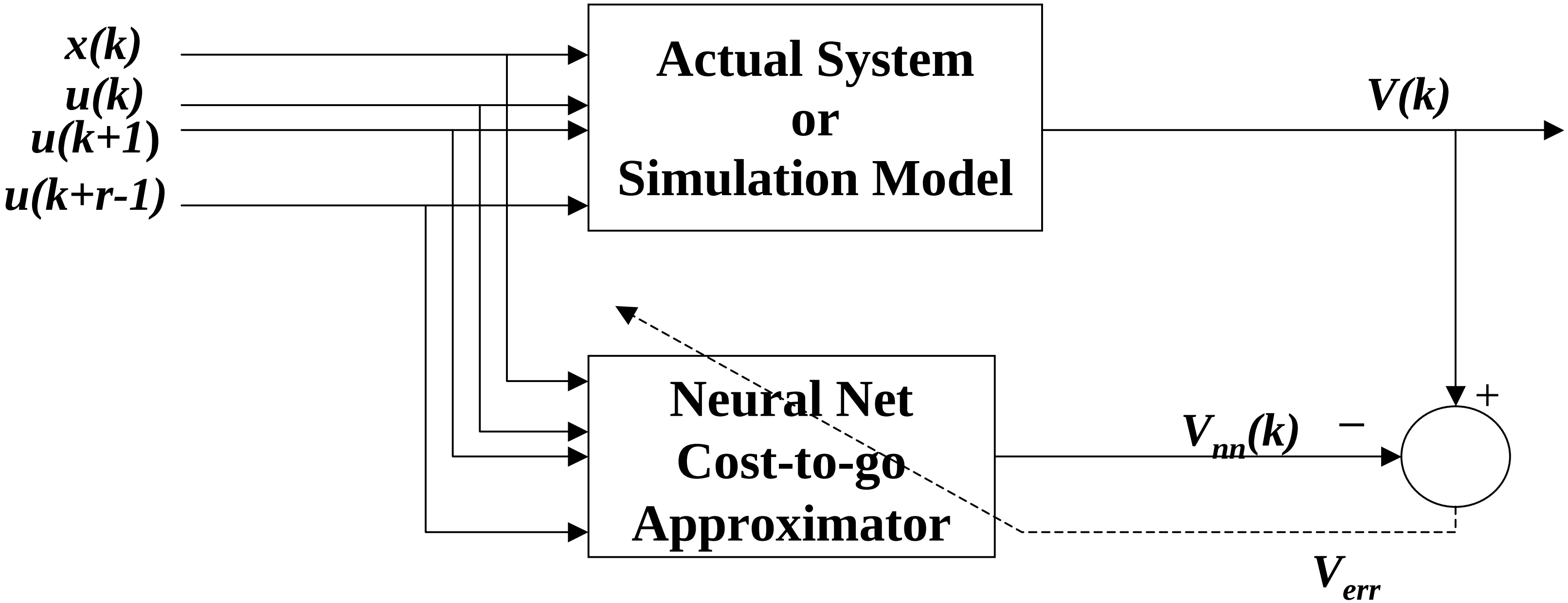
$$x(k),\ u(k), \ldots, u(k+r-1)$$

- Use the analytical model, or a computer simulation or the physical model to generate the future states.
- Use the '*r*' control values and the '*r*' future states to get the ideal cost-to-go function estimate.

$$V(k) = \frac{1}{2}\sum_{i=1}^{r}[x(k+i)^T Q x(k+i) + u(k+i-1)^T R u(k+i-1)]$$

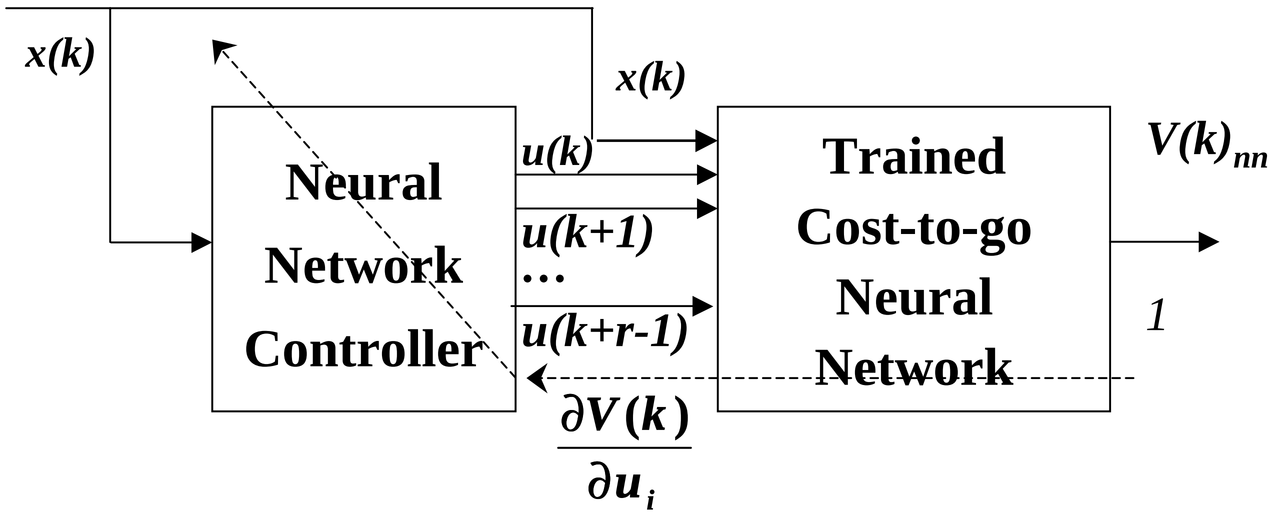
- Use this to train the *CGA* Neural Network

CGA Neural Network Training



Neural Network Cost-to-go Approximator Training

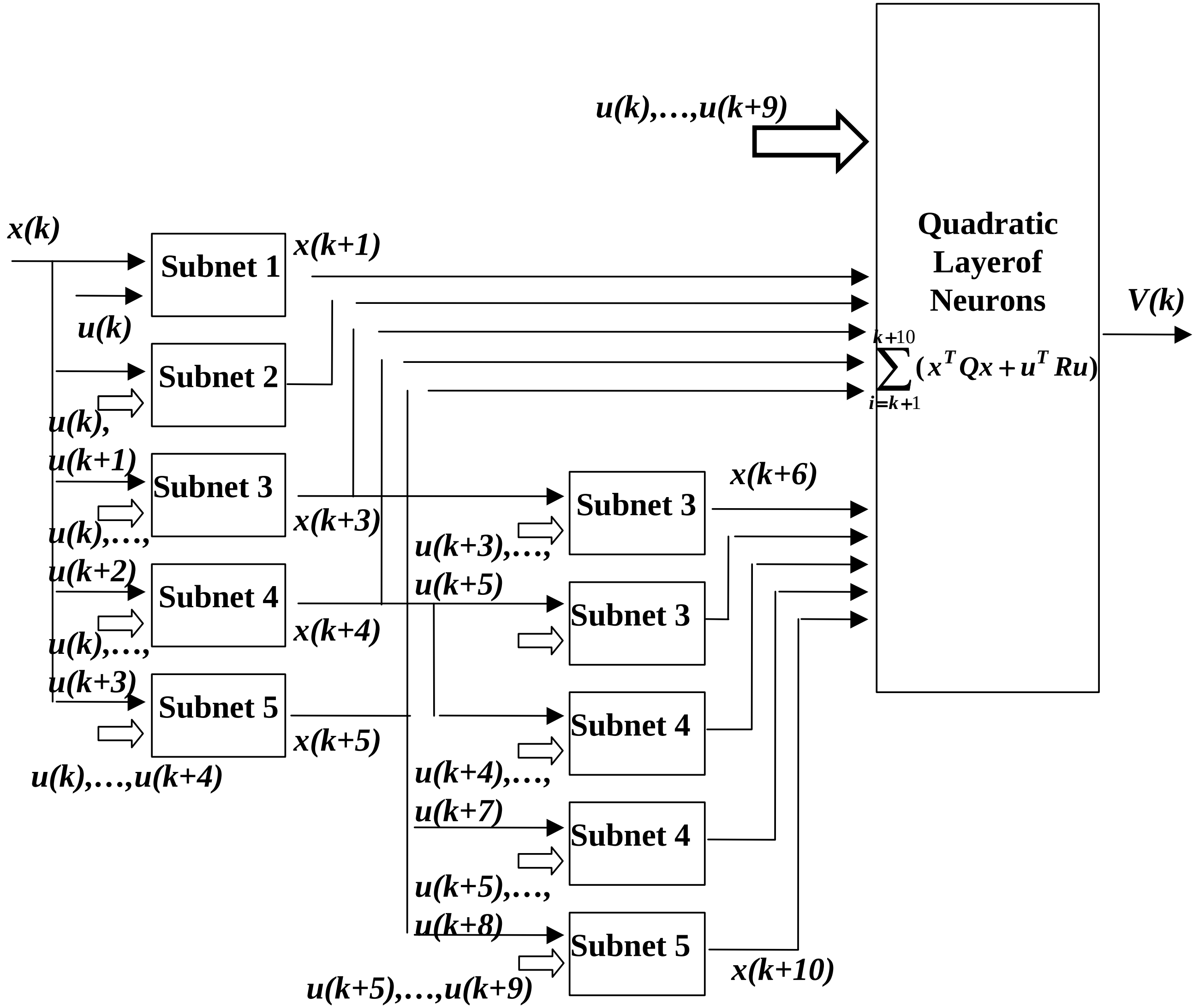
Neural Network Controller Training



- Gradient of *V(k)* with respect to the control inputs *u(k)* ,…, *u(k + r - 1)* is calculated using back-propagation through the '*CGA*' Neural Network.
- These gradients can be further back-propagated through the Neural Network controller to get, $\frac{\partial V(k)}{\partial G_1}$ through $\frac{\partial V(k)}{\partial G_r}$
- Neural Network controller is trained so that

$$\frac{\partial V(k)}{\partial G_i} \rightarrow 0, i = 1 \ldots r$$

Bringing Structure to the *CGA* Network



Implementation of the Hybrid CGA Network of order *'r = 10'*, using trained subnets of order *1* through *5*

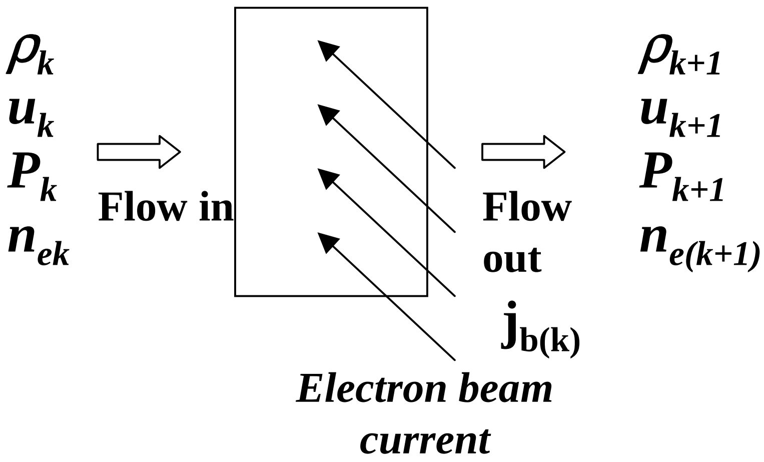
Salient features of the formulation:

- Simplification of the optimization problem
- Decoupled CGA Network training and the controller Network training
- Introduction of structure in the CGA Network
- Same basic architecture for linear or nonlinear systems.
- Data-based implementation - No explicit analytical model needed
- Adaptive control architecture with the use of Neural Networks

Translating the approach to the MHD problem

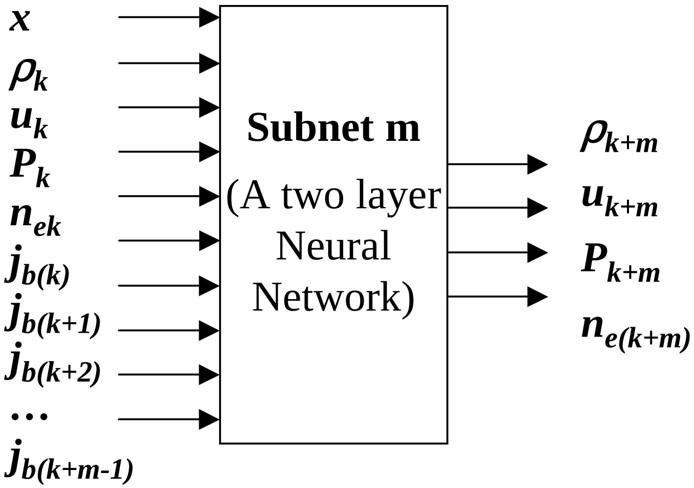
- In terms of the *'x-t'* equivalence, the problem is time-dependent
- Optimization equivalent to the fixed end time optimal control
- Procedure:
  - Defining subnets
  - Parameterizing and training the subnets
  - Arranging them together to get the cost-to-go function *V(0)*
  - Parameterizing and training the Neural Network controller

Defining and Parameterizing the subnets



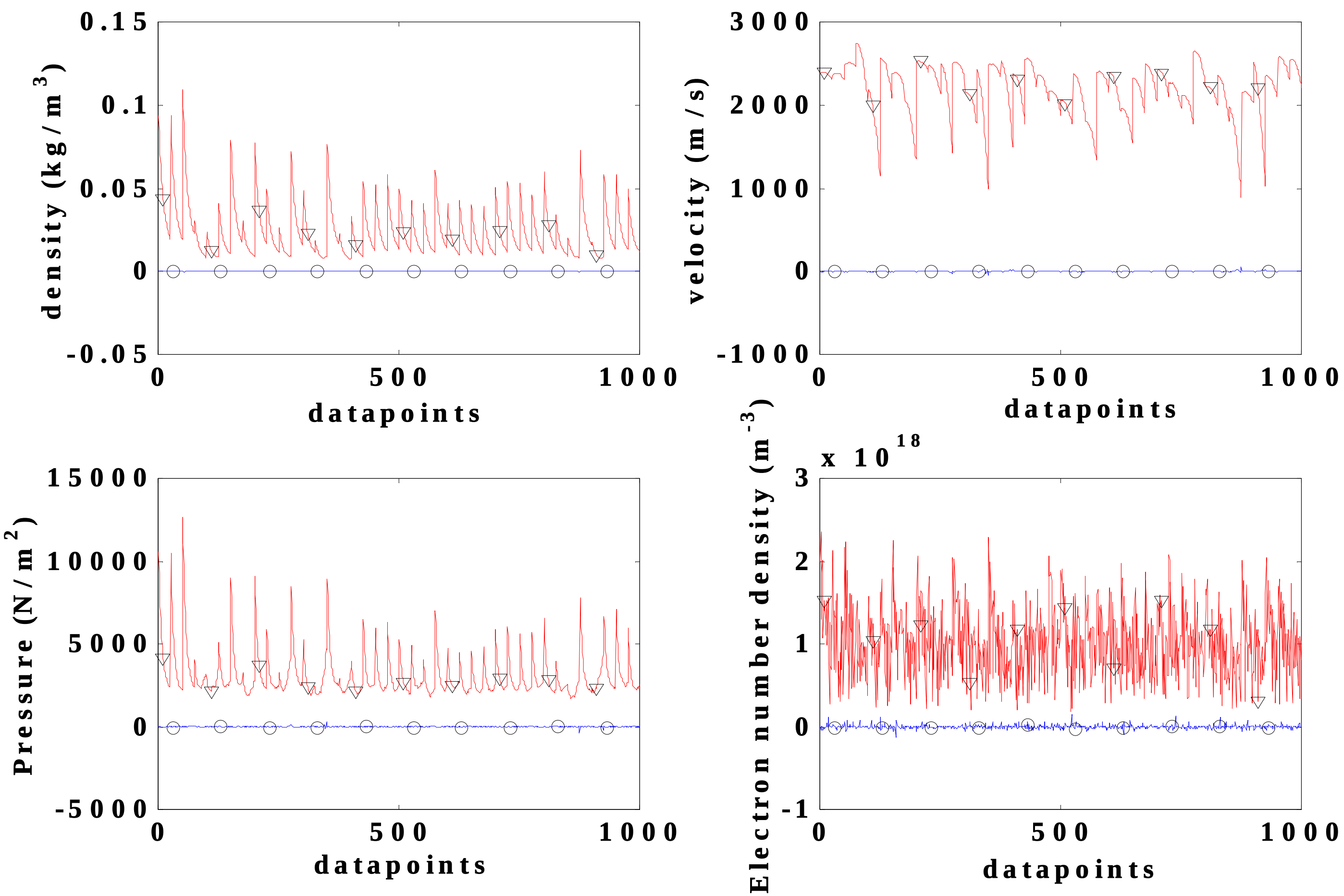
Physical picture describing Subnet 1

- Continuously spaced e-beam windows each having a length of 1 cm
- Subnet 1 chosen to correspond to the system dynamics between a group of 4 e-beam windows
- Length of the channel = 1 m
- Need subnets upto order 25



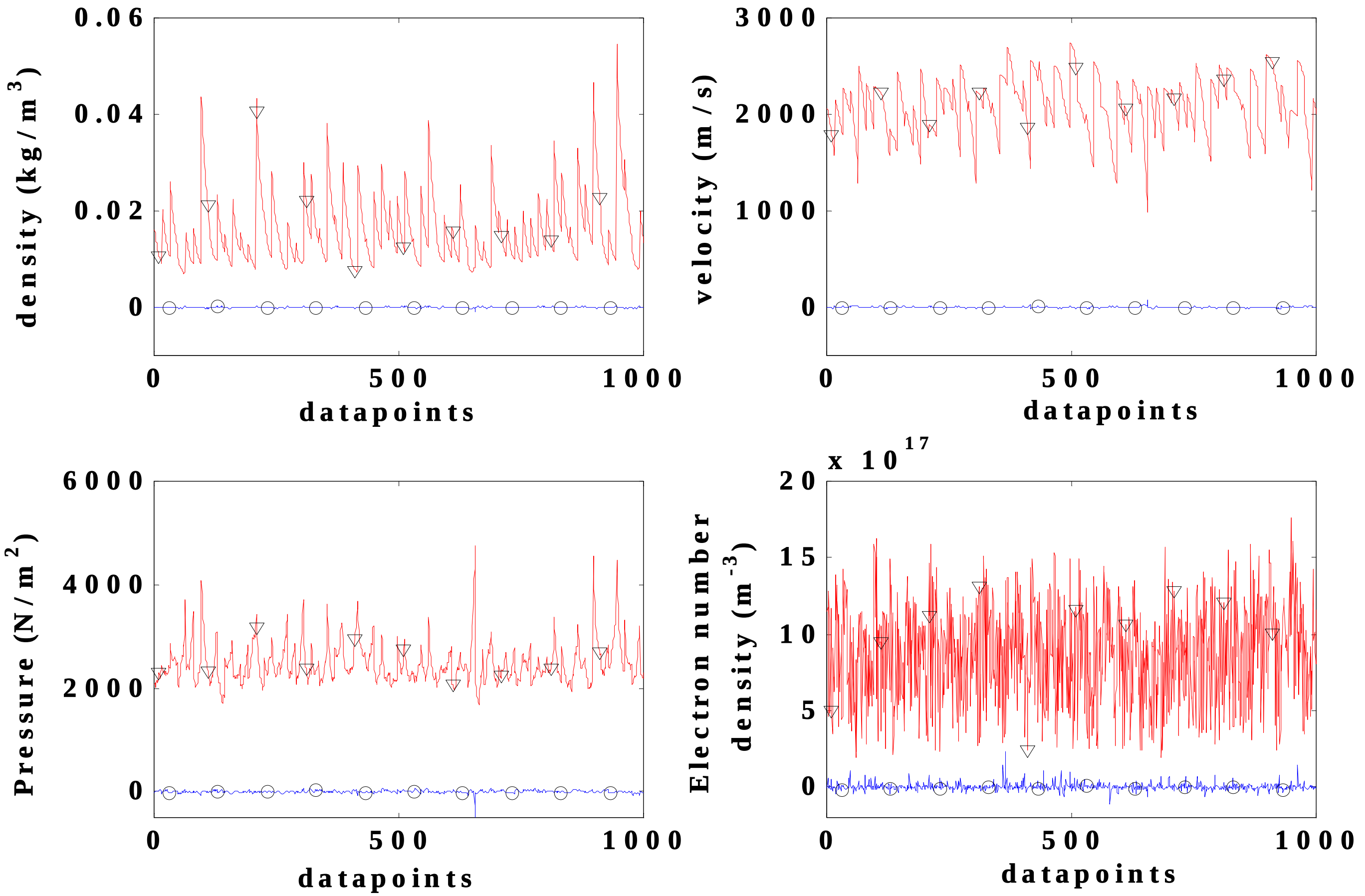
Subnet *m*, inputs and outputs.

Training Results for Subnet 1



Testing Subnet 1, '∇'- Ouput value given by subnet 1, 'o' – Error between the subnet 1 output and the ideal value given by the simulation

Training Results for Subnet 10



Testing Subnet 10, '∇'- Ouput value given by subnet 10, 'o' – Error between the subnet 10 output and the ideal value given by the simulation

Conclusions

- Formulated the problem of performance optimization of the MHD Generator as an optimal control problem
- Implementation in terms of the cost-to-go approach
- Subnets trained upto order *10*
- Higher order subnets built by cascading the trained lower order subnets

Future work

- Arranging the subnets with the fixed Network layers to capture the cost-to-go function.
- Training a Neural Network controller to be optimal.

Conference Papers:

1) Kulkarni, N. V., Phan, M. Q., "Data-Based Cost-To-Go Design for Optimal Control," AIAA Paper No. 2002-4668, AIAA Guidance, Navigation and Control Conference, August 2002.
2) Kulkarni, N. V., Phan, M. Q., "A Neural Networks Based Design of Optimal Controllers for Nonlinear Systems," AIAA Paper No. 2002-4664, AIAA Guidance, Navigation and Control Conference, August 2002.