Optimal Feedback Control of the Magneto-hydrodynamic Generator for a Hypersonic Vehicle

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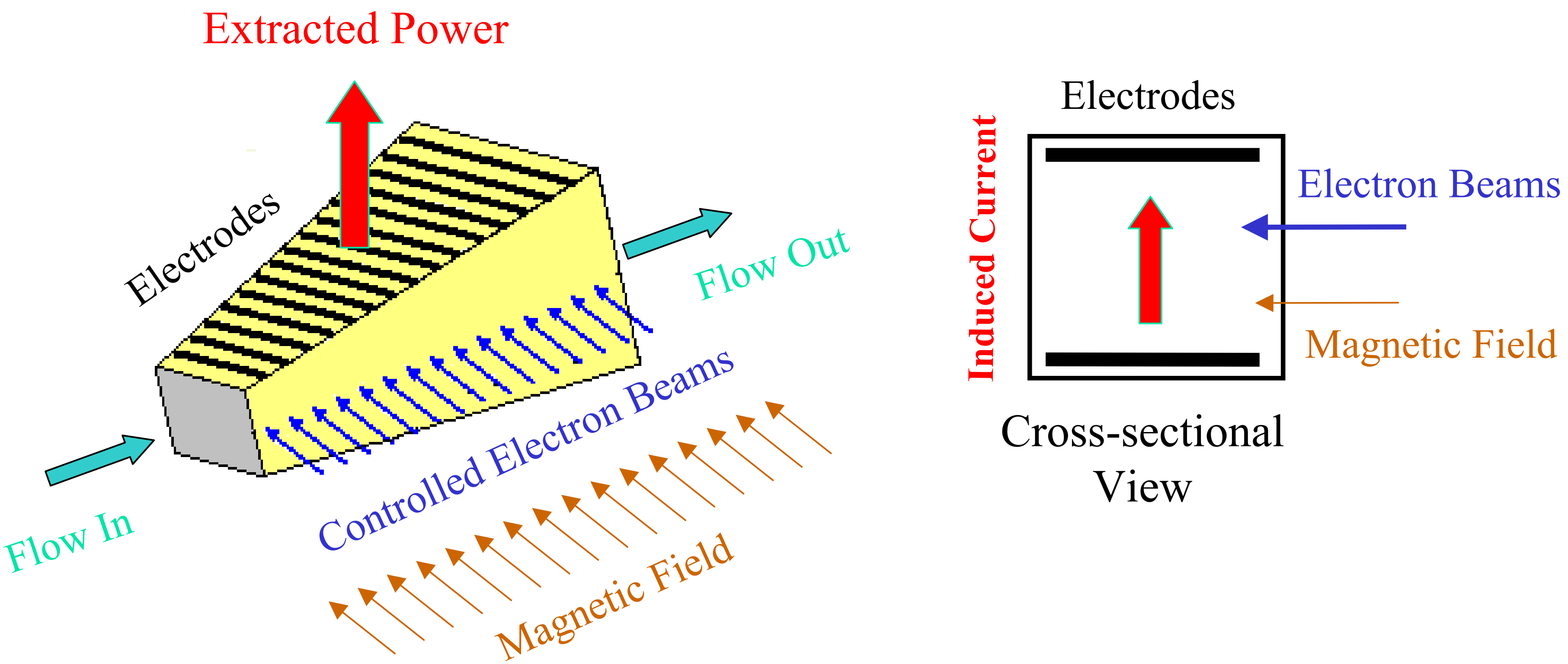
JUP Quarterly Review
April 3-4, 2003
Princeton University, Princeton

Presentation Outline

- The Magneto-hydrodynamic (MHD) generator
- The role of control
- MHD generator system
- Open-loop optimal control architecture
- Dynamic programming based feedback optimal control architecture
- Results
- Conclusions

Magneto-Hydrodynamic (MHD) Generator at the Inlet



Schematic of the MHD Generator

MHD Generator System

- Assumptions
  - One-dimensional steady state flow
  - Inviscid flow
  - No reactive chemistry
  - Low Magnetic Reynolds number
- *x*-*t* equivalence

Flow Equations

- Continuity Equation

$$\frac{d(\rho u A)}{dx} = 0$$

x - Coordinate along the channel

ρ - Fluid density

u - Fluid velocity

A - Channel cross-section area

- Force Equation

$$\rho u \frac{du}{dx} + \frac{dP}{dx} = -(1-k)\sigma u B^2$$

P - Fluid pressure

k - Load factor

σ - Fluid conductivity

B - Magnetic field

Flow Equations…

- Energy Equation

$$\rho u \frac{d(\gamma\varepsilon + \frac{u^2}{2})}{dx} = -k(1-k)\sigma u^2 B^2 + Q_\beta$$

ε - Fluid internal energy

Q_β - Energy deposited by the e-beam

- Continuity Equation for the electron number density

$$\frac{d(n_e u)}{dx} = \frac{2 j_b \varepsilon_b}{eY_i Z} - \beta n_e^2$$

n_e - Electron number density

j_b - Electron beam current

ε_b - E-beam energy

Z - Channel width

Y - Ionization potential

Performance Characterization

$$J = p_1\left[T(x_f) - T_e\right]^2 + p_2\left[M(x_f) - M_e\right]^2 + \\ + \int_0^{x_f} \left[\begin{array}{l} \dfrac{q_1}{\rho u A}\left[Q_\beta A - k(1-k)\sigma u^2 B^2 A\right] + \\ q_2 h(P) + q_3 dS^2 + r_1 j_b{}^2 \end{array} \right] dx$$

- Attaining prescribed values of flow variables at the channel exit (Mach number, Temperature)
- Maximizing the net energy extracted which is the difference between the energy extracted and the energy spent on the e-beam ionization
- Minimizing adverse pressure gradients
- Minimizing the entropy rise in the channel
- Minimizing the use of excessive electron beam current

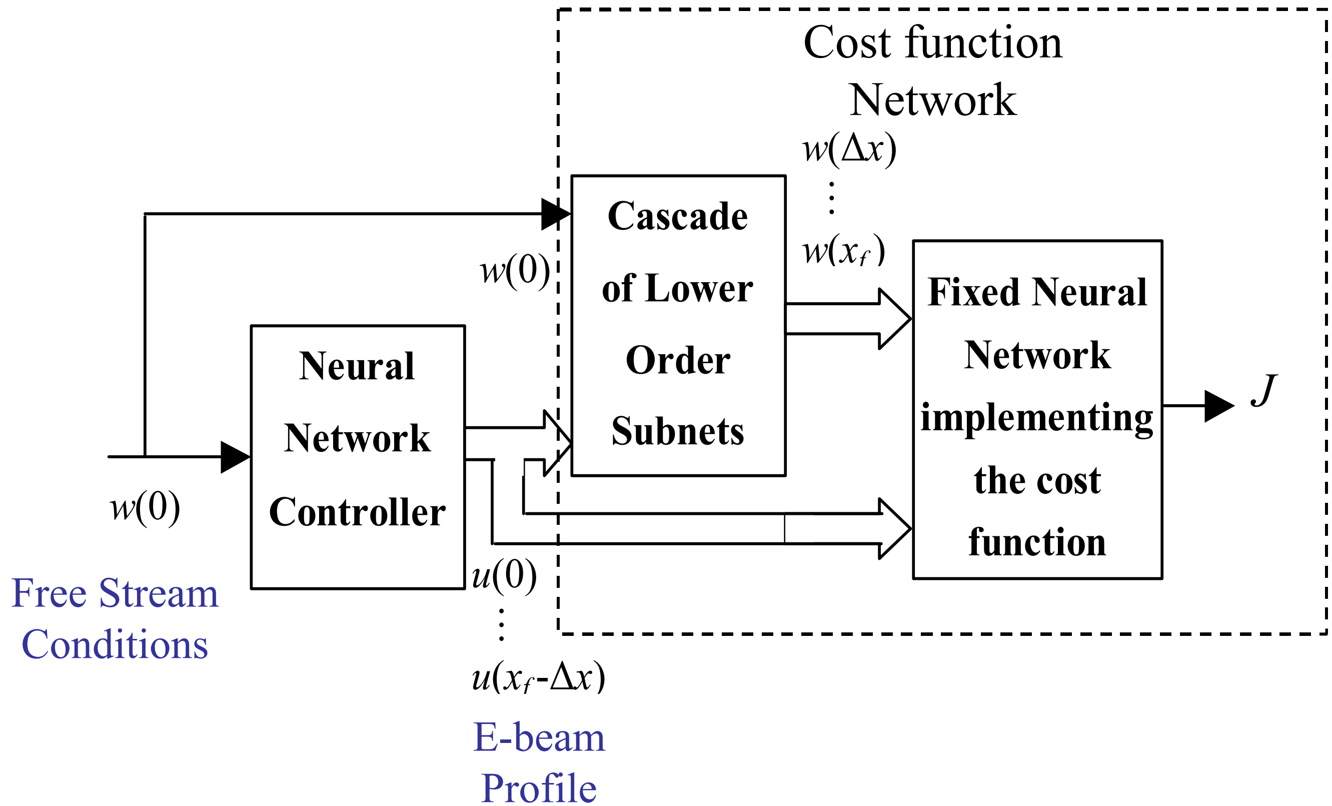
The Predictive Control Based Approach for Optimal Control

- Features of our optimal controller design technique
  - Works for both linear and nonlinear systems
  - Data-based
  - Finite horizon, end-point optimal control problem
  - Equivalent to time (position) varying system dynamics

[1] Kulkarni, N.V. and Phan, M.Q., "Data-Based Cost-To-Go Design for Optimal Control," *AIAA Paper* 2002-4668, *AIAA Guidance, Navigation and Control Conference*, August 2002.

[2] Kulkarni, N.V. and Phan, M.Q., "A Neural Networks Based Design of Optimal Controllers for Nonlinear Systems," *AIAA Paper* 2002-4664, *AIAA Guidance, Navigation and Control Conference*, August 2002.

Open Loop Optimal Control Using Neural Networks



Optimal control architecture

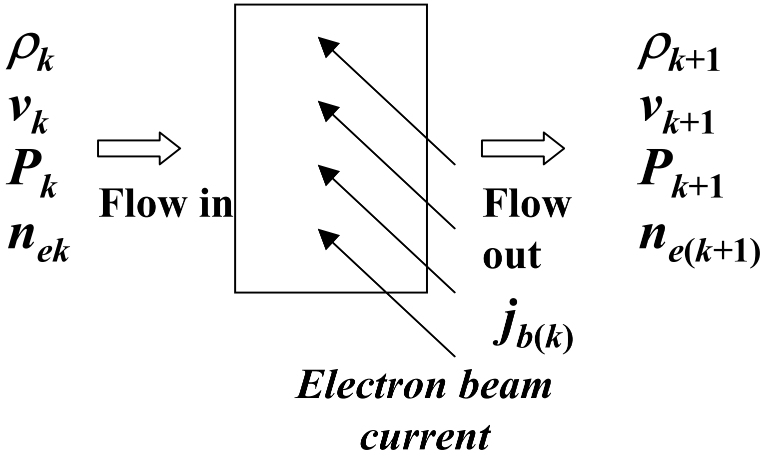
[3] Kulkarni, N.V. and Phan, M.Q., "Performance Optimization of a Magneto-hydrodynamic Generator at the Scramjet Inlet," *AIAA Paper* 2002-5121, 11th *AIAA/AAAF International Spaceplanes and Hypersonic Technologies Conference*, Sept. 2002.

Formulation of the Control Architecture: Cost Function Approximator

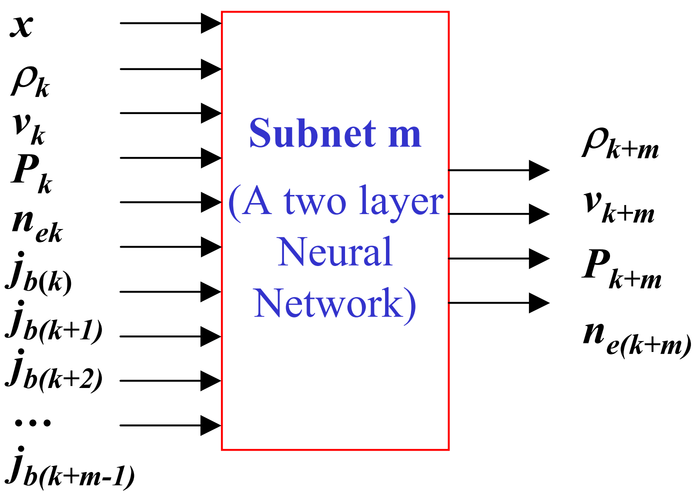
- Collecting system data through simulation or a physical model
- Parameterizing single step ahead and multi-step ahead models called subnets using neural networks
- Training the subnets using system data
- Formulating a fixed layer neural network that take the subnet outputs and calculate the cost-to-go function or the cumulative cost function.

Using Subnets to Build the Cost Function Network

- Continuously spaced e-beam windows each having a length of 0.5 cm
- Subnet 1 chosen to correspond to the system dynamics between a group of 4 e-beam windows
- Length of the channel = 1 m
- Need subnets up to order 50

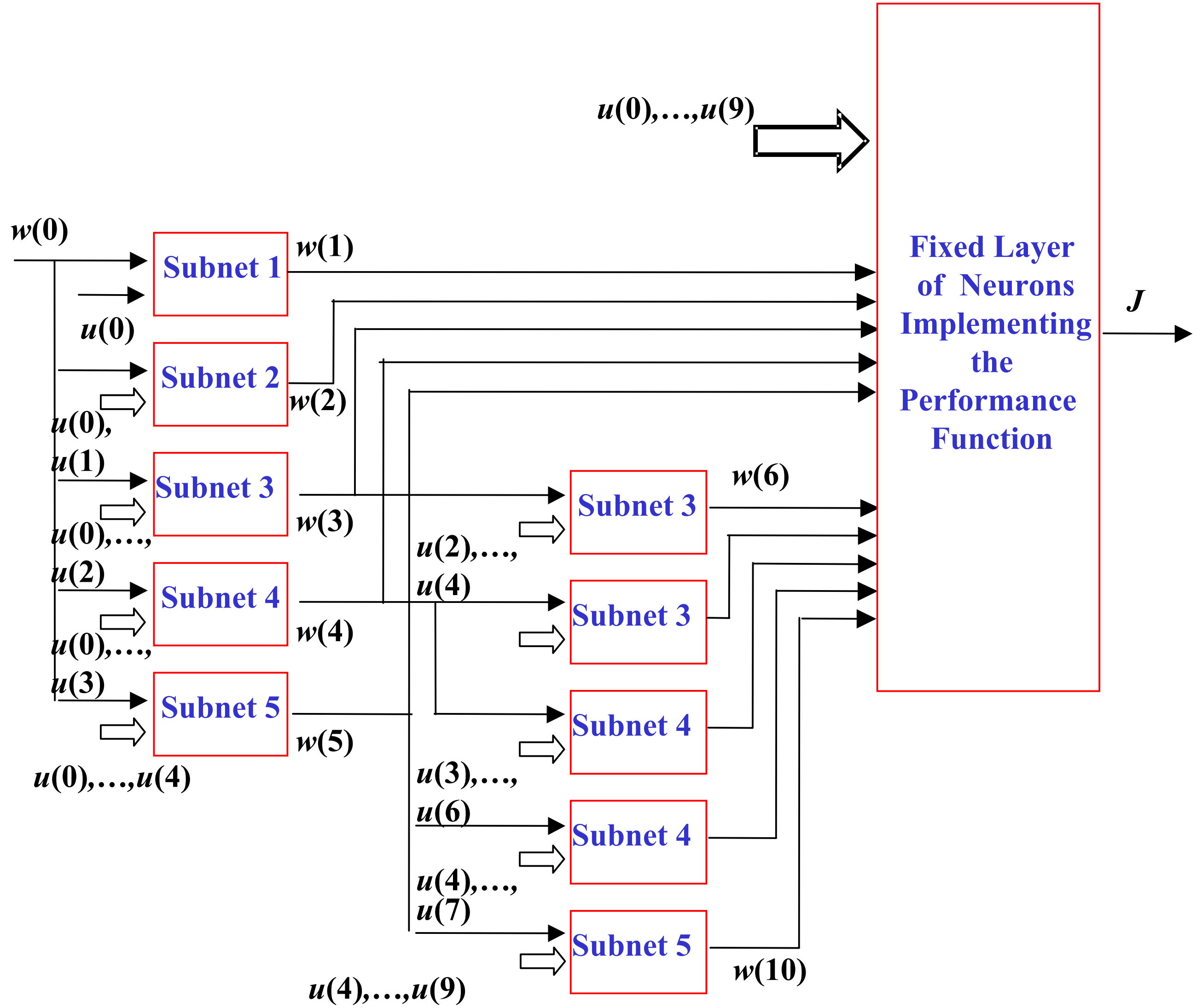


Physical picture describing Subnet 1



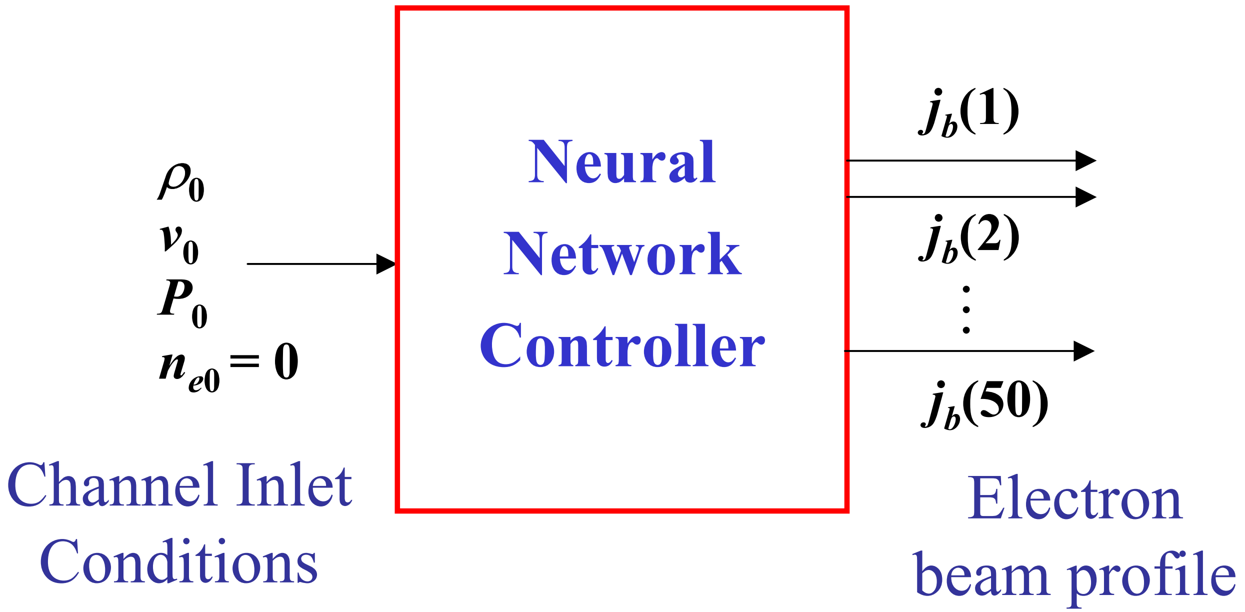
Subnet *m*, inputs and outputs.

Cost Function Network

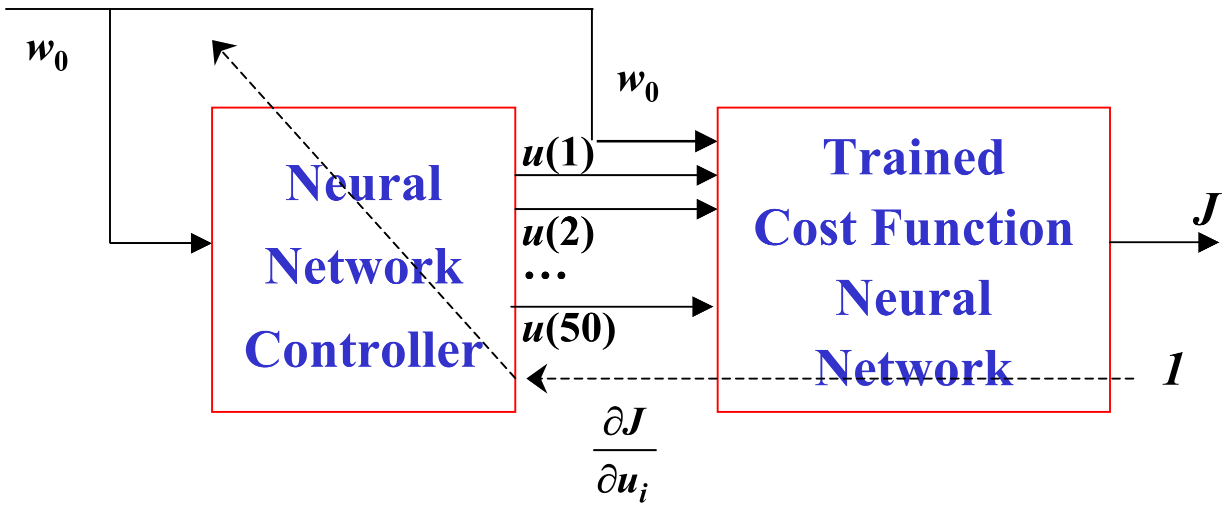


Implementation of the Cost function network of order r = 10, using trained subnets of order 1 through 5

Formulation of the Control Architecture: Neural Network Controller



Neural Network Controller Training



- Gradient of J with respect to the control inputs $u(1)$,…, $u(50)$ is calculated using back-propagation through the *CGA* neural network.
- These gradients can be further back-propagated through the neural network controller to get, $\frac{\partial J}{\partial W_{nn}}$, W_{nn} - weights of the network)
- Neural network controller is trained so that

$$\frac{\partial J}{\partial W_{nn}} \rightarrow 0$$

Difficulties with the Open-Loop Approach

- Computational complexity
- Need for a feedback solution

Dynamic Programming based State Feedback Control

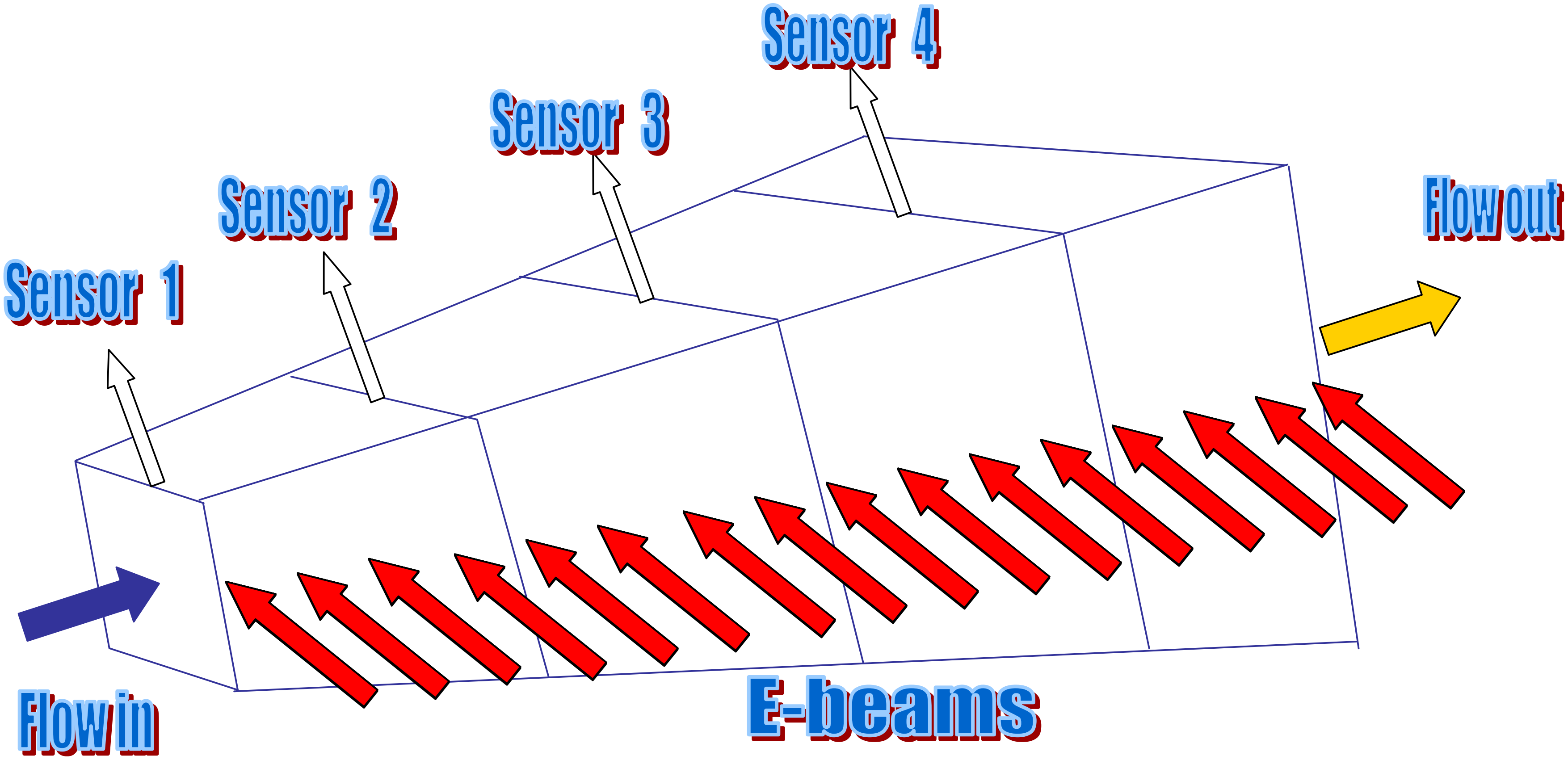
- Using the dynamic programming principle to design the controllers along the channel

An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

- Richard Bellman

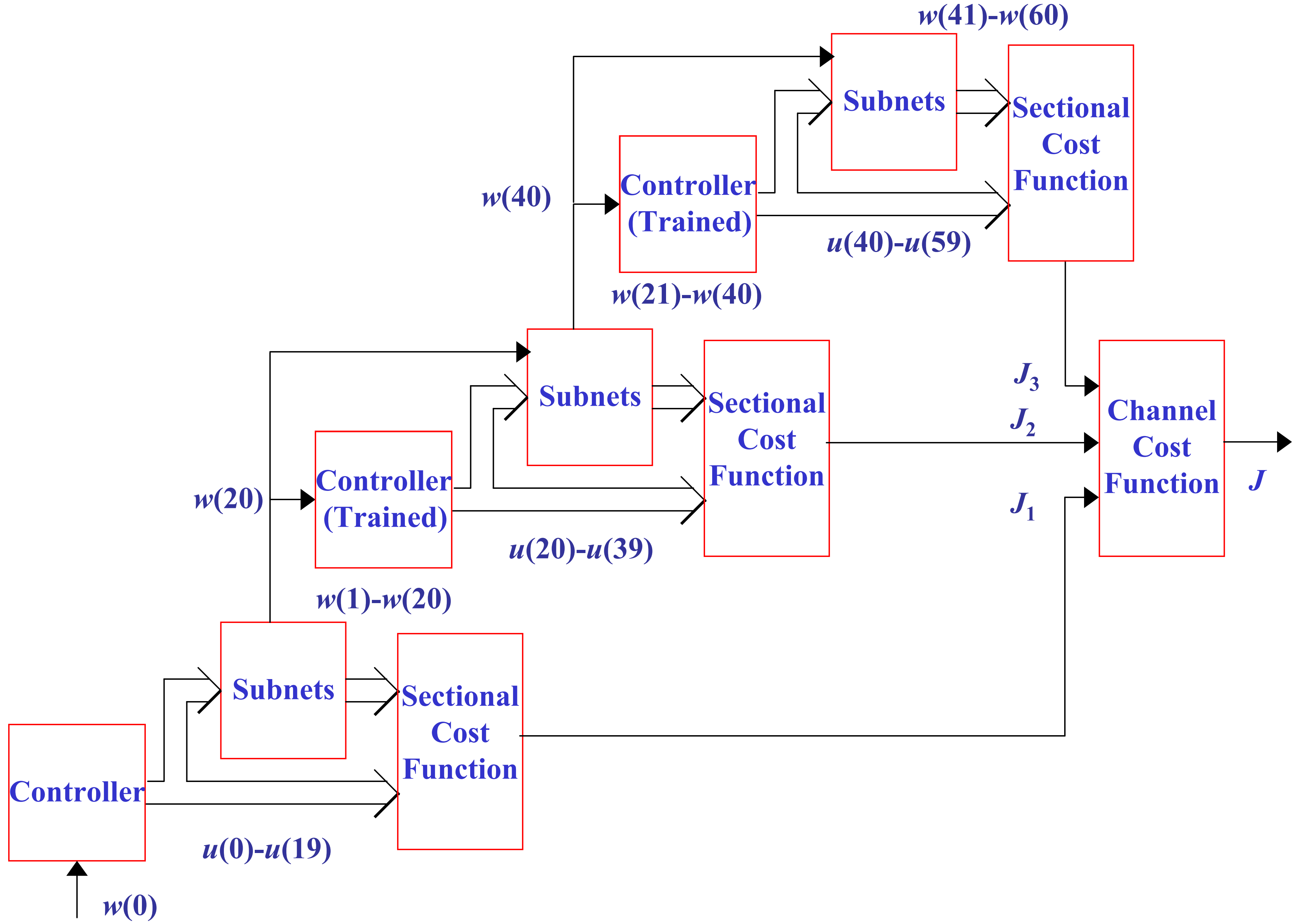
- Assume available sensors along the channel

Dynamic Programming Based State-Feedback Architecture



Inlet MHD Channel

Details of the Control Architecture



Channel Geometry

- Length - 1.6 m
- Square cross-section
- Inlet width - 0.15 m
- Exit width - 0.3 m
- Electron beam windows continuously distributed
- Electron beam window width - 0.5 cm
- Groups of 4 electron beam windows with same control applied (total of 80 control e-beam values)
- Four sets of sensors evenly spaced along the channel

Maximizing the Net Energy Extracted while Achieving a Prescribed Exit Mach Number

Cost function:

$$J = p_1\left[T(x_f) - T_e\right]^2 + p_2\left[M(x_f) - M_e\right]^2 +$$

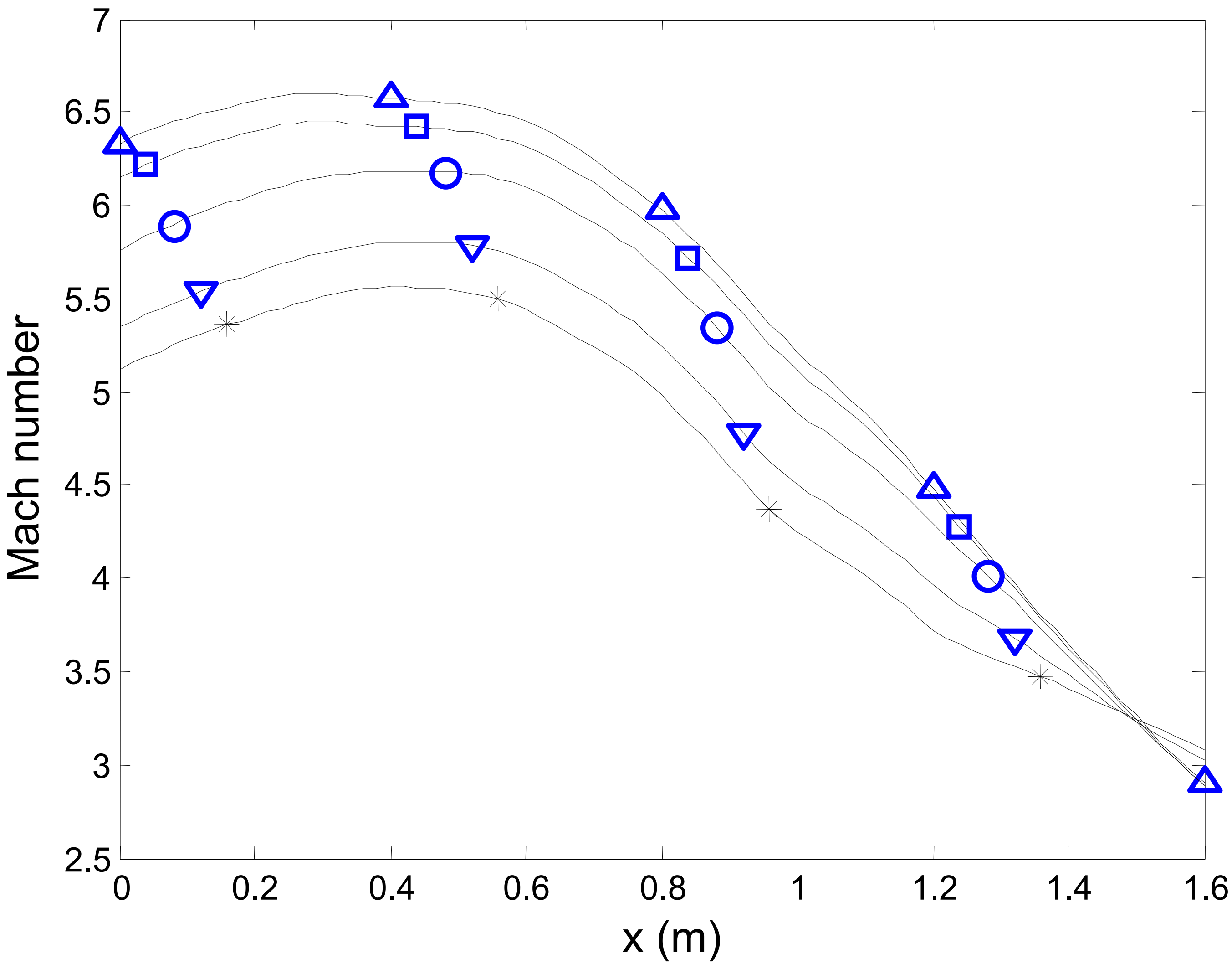
$$+\sum_{i=1}^{80}\left[\begin{array}{l}\dfrac{q_1}{\rho(i)u(i)A(i)}\left[Q_\beta(i)A(i) - k(1-k)\sigma(i)u(i)^2B^2A(i)\right] + \\ q_2(i)P(i) + q_3[S(i) - S(i-1)]^2 + r_1 j_b(i-1)^2\end{array}\right]\Delta x$$

p_1	p_2	q_1	q_2	q_3	r_1
20	0	0.00001	0	0	0.005

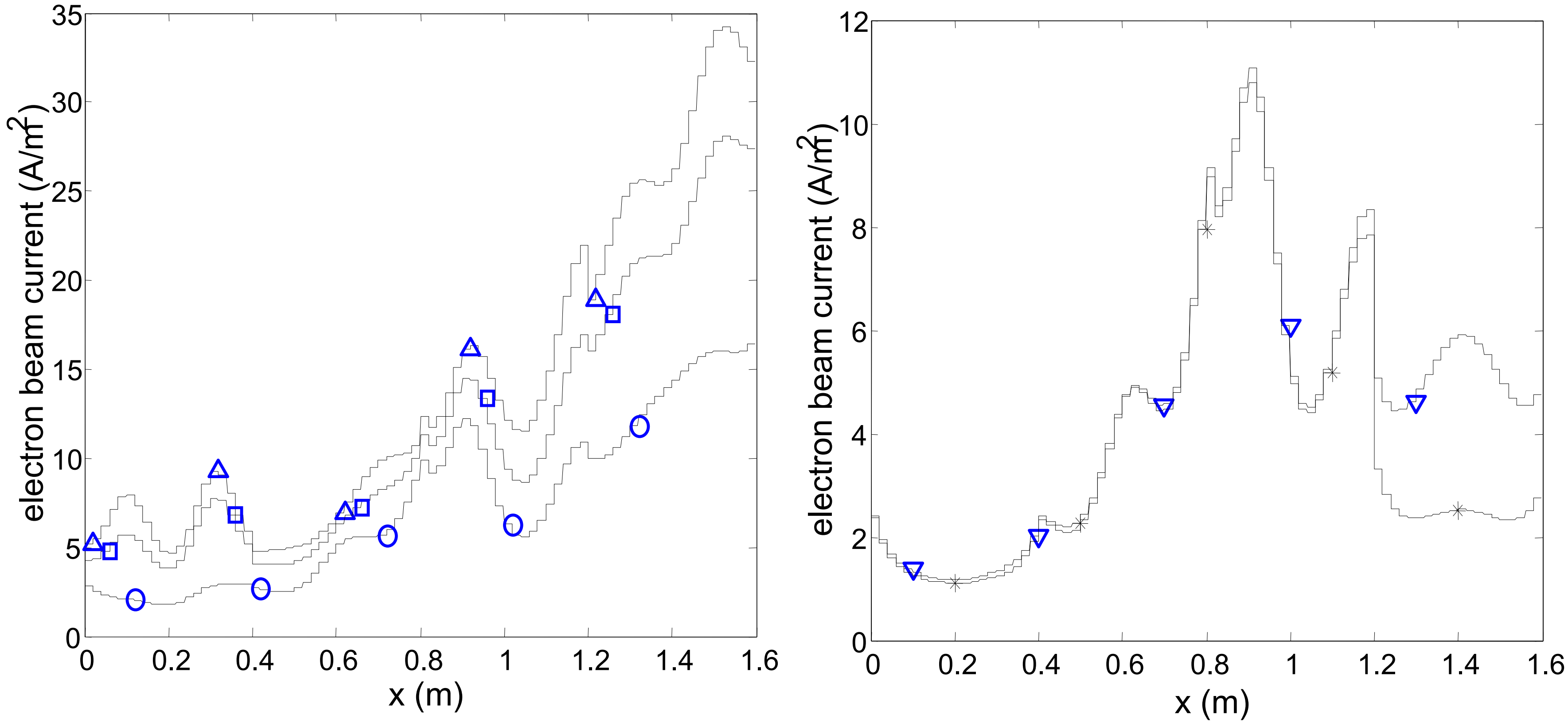
$$M_e = 3$$

Exit Mach number for different initial free stream conditions

Free Stream Altitude	Free Stream Mach number	Exit Mach number	Legend in the plots
30 km	9.2	2.90	——— Δ
30 km	8.8	2.89	- - - - □
30 km	8	2.90	…… O
30 km	7.2	3.02	-.-.-.- ∇
30 km	6.8	3.08	——— *



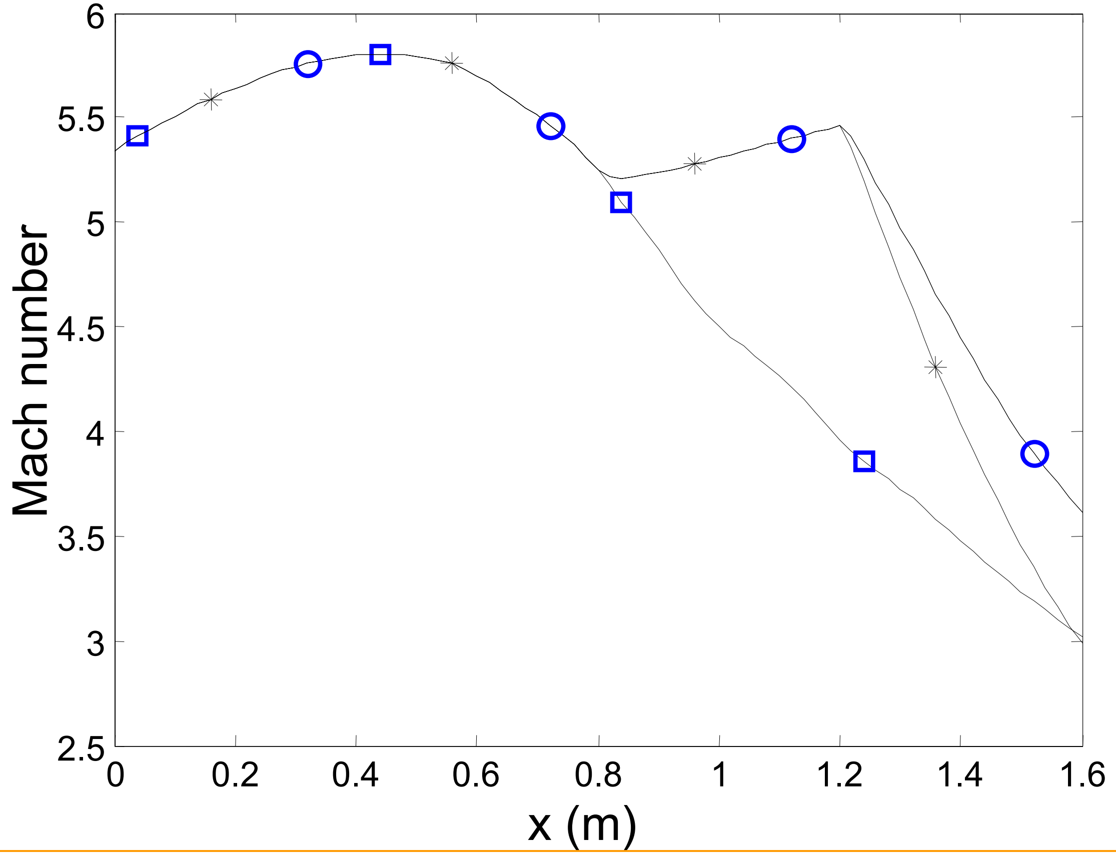
Mach number profiles for different free stream conditions.
Refer to Table for legend



Electron beam current profiles for different free stream conditions.
Refer to Table 5 for legend

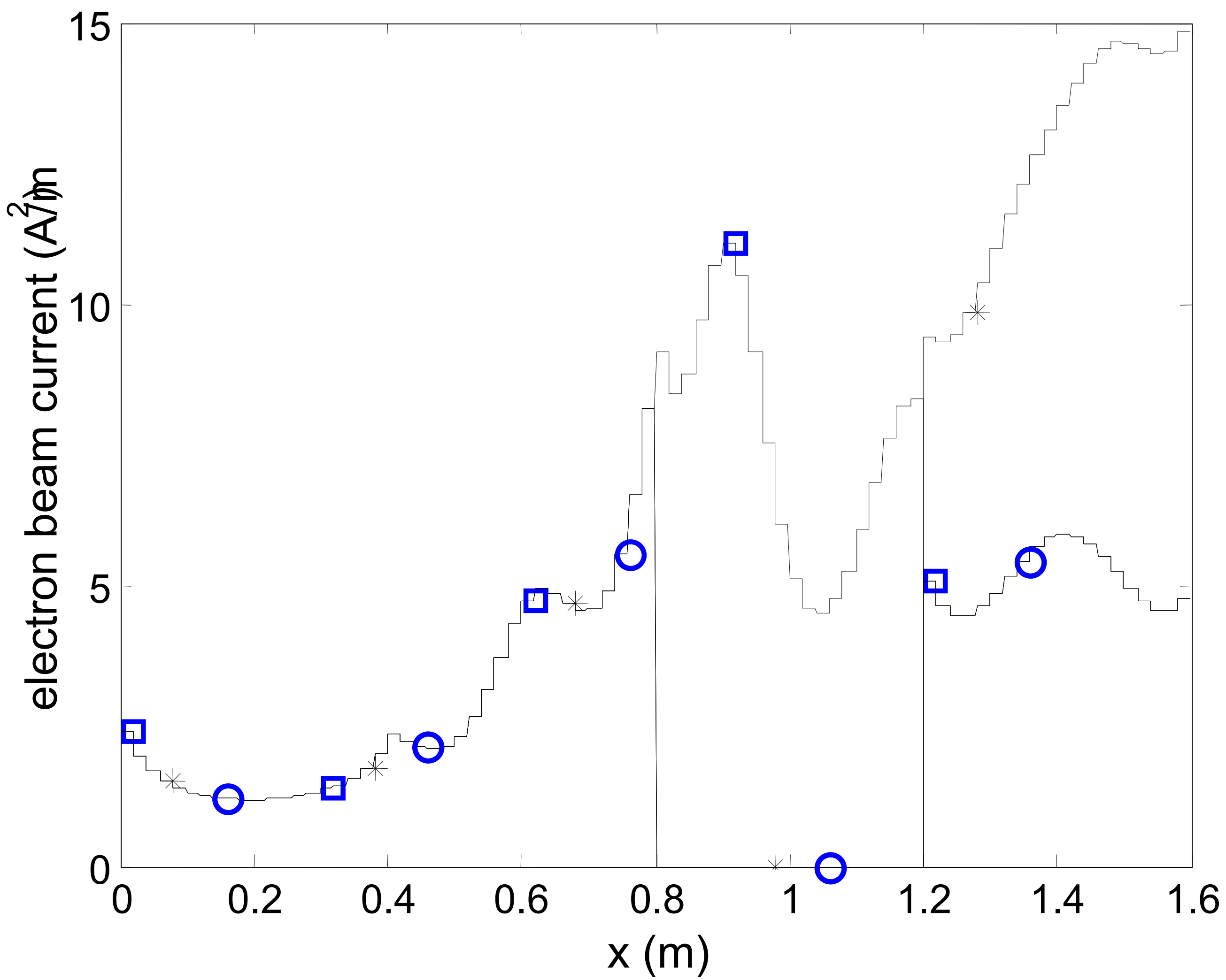
Feedback nature of the control architecture

- Assume that the control actuators between the location of sensor 3 and sensor 4 have failed.
- Performance measure : Maximize the energy extracted while achieving the prescribed exit Mach number, $M_e = 3$



Mach number profiles

□ – with no failure,
O – open-loop with failure,
* – feedback with failure



Control profiles to illustrate feedback approach.

□ - with no failure, O – open-loop with failure, * - feedback with failure

Conclusions

- A neural networks based optimal controller design for the MHD channel
- Data-based approach.
- A closed-loop optimal control solution based on the principle of dynamic programming
- Successful illustration of the feedback nature of the control approach for failed actuator case.