Aerodynamic Shape Optimization Techniques Based On Control Theory

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OBJECTIVE OF COMPUTATIONAL AERODYNAMICS

1. Capability to predict the flow past an airplane in different flight regimes
   - take off
   - cruise (transonic)
   - flutter
2. Interactive design calculations to allow immediate improvement
3. Automatic design optimization

AERODYNAMIC DESIGN METHODS

1945 LIGHTHILL (Conformal Mapping, Incompressible Flow)

1965 NIEUWLAND (Hodograph, Power Series)

1970 GARABEDIAN - KORN (Hodograph, Complex Characteristics)

1974 BOERSTOEL (Hodograph)

1974 TRENEN (Potential Flow, Dirichlet Boundary Conditions)

1977 HENNE (3-D Potential Flow, Based on FLO22)

1985 VOLPE-MELNIK (2-D Potential Flow, Based on FLO36)

1979 GARABEDIAN - McFADDEN (Potential Flow, Neuman Boundary Conditions, Iterated Mapping)

1976 SOBIECZI (Fictitious Gas)

1979 DRELA - GILES (2-D Euler Equations, Streamline Coordinates, Newton Iteration)

TRADITIONAL APPROACH TO DESIGN OPTIMIZATION

The simplest approach to optimization is to define the geometry through a set of design parameters, which may, for example, be the weights α_i applied to a set of shape functions $b_i(x)$ so that the shape is represented as

$$f(x) = \sum \alpha_i b_i(x).$$

Then a cost function I is selected. The sensitivities $\frac{\partial I}{\partial \alpha_i}$ may now be estimated by making a small variation $\delta\alpha_i$ in each design parameter in turn and recalculating the flow to obtain the change in I. Then

$$\frac{\partial I}{\partial \alpha_i} \approx \frac{I(\alpha_i + \delta\alpha_i) - I(\alpha_i)}{\delta\alpha_i}.$$

The gradient vector $\frac{\partial I}{\partial \alpha}$ may now be used to determine a direction of improvement.

The simplest procedure is to make a step in the negative gradient direction by setting

$$\alpha^{n+1} = \alpha^{n} - \lambda \delta \alpha,$$

so that to first order

$$I + \delta I = I - \frac{\partial I^T}{\partial \alpha} \delta \alpha = I - \lambda \frac{\partial I^T}{\partial \alpha} \frac{\partial I}{\partial \alpha}.$$

DISADVANTAGES

The main disadvantage of this approach is the need for a number of flow calculations proportional to the number of design variables to estimate the gradient. The computational costs can thus become prohibitive as the number of design variables is increased.

GENERAL FORMULATION OF THE ADJOINT APPROACH TO OPTIMAL DESIGN

The progress of the design procedure is measured in terms of a cost function

$$I = I\left(w, \mathcal{F}\right),$$

where w are the flow-field variables and $\mathcal{F}$ is the location of the boundary.

A change in $\mathcal{F}$ results in a change

$$\delta I = \left[\frac{\partial I^T}{\partial w}\right]_I \delta w + \left[\frac{\partial I^T}{\partial \mathcal{F}}\right]_{II} \delta \mathcal{F}, \tag{3}$$

in the cost function. [1]

[1] the subscripts I and II are used to distinguish the contributions due to the variation δw in the flow solution from the change associated directly with the modification $\delta \mathcal{F}$ in the shape

The governing equations of the flow field

$$R\left(w,\mathcal{F}\right)=0. \tag{4}$$

which express the dependence of w and $\mathcal{F}$ are introduced as a constraint within the flow-field domain $\mathcal{D}$. Then δw is determined from the equation

$$\delta R=\left[\frac{\partial R}{\partial w}\right]_I \delta w+\left[\frac{\partial R}{\partial \mathcal{F}}\right]_{II} \delta \mathcal{F}=0. \tag{5}$$

Next, introducing a Lagrange Multiplier ψ, we have

$$\begin{aligned}
\delta I &= \frac{\partial I^T}{\partial w}\delta w+\frac{\partial I^T}{\partial \mathcal{F}}\delta \mathcal{F}-\psi^T\left(\left[\frac{\partial R}{\partial w}\right]\delta w+\left[\frac{\partial R}{\partial \mathcal{F}}\right]\delta \mathcal{F}\right)\\
&= \left\{\frac{\partial I^T}{\partial w}-\psi^T\left[\frac{\partial R}{\partial w}\right]\right\}_I \delta w+\left\{\frac{\partial I^T}{\partial \mathcal{F}}-\psi^T\left[\frac{\partial R}{\partial \mathcal{F}}\right]\right\}_{II} \delta \mathcal{F}.
\end{aligned} \tag{6}$$

Choosing ψ to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w} \tag{7}$$

The first term is eliminated, and we find that

$$\delta I = \mathcal{G}\delta\mathcal{F}, \tag{8}$$

where

$$\mathcal{G} = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right].$$

Once equation (8) is established, an improvement can be made with a shape change

$$\delta\mathcal{F} = -\lambda\mathcal{G}$$

where λ is positive, and small enough that the first variation is an accurate estimate of δI. The variation in the cost function then becomes

$$\delta I = -\lambda\mathcal{G}^T\mathcal{G} < 0.$$

ADVANTAGES

- Equation (8) is independent of δw. Hence, the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations.
- The computational cost of a single design cycle is roughly equivalent to the cost of two flow solutions since the the adjoint problem has similar complexity.
- When the number of design variables becomes large, the computational efficiency of the control theory approach over traditional approach, which requires direct evaluation of the gradients by individually varying each design variable and recomputing the flow field, becomes compelling.

DESIGN USING THE EULER EQUATIONS

Denote the Cartesian coordinates and velocity components by x_1, x_2, x_3 and u_1, u_2, u_3, and use the convention that summation over $i = 1$ to 3 is implied by a repeated index i. Then, the three-dimensional Euler equations may be written as

$$\frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = 0 \quad \text{in } D, \tag{32}$$

where

$$w = \left\{ \begin{array}{c} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{array} \right\}, \quad f_i = \left\{ \begin{array}{c} \rho u_i \\ \rho u_i u_1 + p\delta_{i1} \\ \rho u_i u_2 + p\delta_{i2} \\ \rho u_i u_3 + p\delta_{i3} \\ \rho u_i H \end{array} \right\} \tag{33}$$

and δ_{ij} is the Kronecker delta function. Also,

$$p = (\gamma - 1)\,\rho\left\{E - \frac{1}{2}\left(u_i^2\right)\right\}, \tag{34}$$

and

$$\rho H = \rho E + p \tag{35}$$

where γ is the ratio of the specific heats.

Consider a transformation to coordinates ξ_1, ξ_2, ξ_3 where

$$K_{ij} = \left[\frac{\partial x_i}{\partial \xi_j}\right], \quad J = \det(K), \quad K_{ij}^{-1} = \left[\frac{\partial \xi_i}{\partial x_j}\right],$$

and

$$Q = JK^{-1}.$$

The elements of Q are the coefficients of K, and in a finite volume discretization they are just the face areas of the computational cells projected in the x_1, x_2, and x_3 directions.

Also introduce scaled contravariant velocity components as

$$U_i = Q_{ij} u_j.$$

The Euler equations can now be written as

$$\frac{\partial W}{\partial t} + \frac{\partial F_i}{\partial \xi_i} = 0 \quad \text{in } D, \tag{36}$$

where

$$W = Jw,$$

and

$$F_i = Q_{ij} f_j = \begin{bmatrix} \rho U_i \\ \rho U_i u_1 + Q_{i1} p \\ \rho U_i u_2 + Q_{i2} p \\ \rho U_i u_3 + Q_{i3} p \\ \rho U_i H \end{bmatrix}.$$

Assume now that the new computational coordinate system conforms to the wing in such a way that the wing surface B_W is represented by $\xi_2 = 0$.

Then the flow is determined as the steady state solution of equation (36) subject to the flow tangency condition

$$U_2 = 0 \quad \text{on } B_W. \tag{37}$$

At the far field boundary B_F, conditions are specified for incoming waves, as in the two-dimensional case, while outgoing waves are determined by the solution.

The weak form of the Euler equations for steady flow can be written as

$$\int_{\mathcal{D}} \frac{\partial \phi^T}{\partial \xi_i} F_i d\mathcal{D} = \int_{\mathcal{B}} n_i \phi^T F_i d\mathcal{B}, \tag{38}$$

where the test vector ϕ is an arbitrary differentiable function and n_i is the outward normal at the boundary. If a differentiable solution w is obtained to this equation, it can be integrated by parts to give

$$\int_{\mathcal{D}} \phi^T \frac{\partial F_i}{\partial \xi_i} d\mathcal{D} = 0.$$

Suppose now that it is desired to control the surface pressure by varying the wing shape. For this purpose, it is convenient to retain a fixed computational domain and then, variations in the shape result in corresponding variations in the mapping derivatives defined by K. Introduce the cost function

$$I = \frac{1}{2} \iint_{B_W} (p - p_d)^2 \, d\xi_1 d\xi_3,$$

where p_d is the desired pressure. The design problem is now treated as a control problem where the control function is the wing shape, which is to be chosen to minimize I subject to the constraints defined by the flow equations (36–). A variation in the shape will cause a variation δp in the pressure and consequently a variation in the cost function

$$\delta I = \iint_{B_W} (p - p_d) \, \delta p \;\; d\xi_1 d\xi_3. \tag{39}$$

Since p depends on w through the equation of state (34–35), the variation δp can be determined from the variation δw. Define the Jacobian matrices

$$A_i = \frac{\partial f_i}{\partial w}, \qquad C_i = Q_{ij} A_j. \tag{40}$$

The weak form of the equation for δw in the steady state becomes

$$\int_{\mathcal{D}} \frac{\partial \phi^T}{\partial \xi_i} \delta F_i d\mathcal{D} = \int_{\mathcal{B}} (n_i \phi^T \delta F_i) d\mathcal{B},$$

where

$$\delta F_i = C_i \delta w + \delta Q_{ij} f_j,$$

which should hold for any differential test function ϕ. This equation may be added to the variation in the cost function, which may now be written as

$$\delta I = \iint_{B_W} (p - p_d)\, \delta p \;\; d\xi_1 d\xi_3 - \int_{\mathcal{D}} \left(\frac{\partial \psi^T}{\partial \xi_i} \delta F_i \right) d\mathcal{D} + \int_{\mathcal{B}} \left(n_i \psi^T \delta F_i \right) d\mathcal{B}. \tag{41}$$

On the wing surface B_W, $n_1 = n_3 = 0$. Thus, it follows from equation (37) that

$$\delta F_2 = \begin{bmatrix} 0 \\ Q_{21}\delta p \\ Q_{22}\delta p \\ Q_{23}\delta p \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \delta Q_{21} p \\ \delta Q_{22} p \\ \delta Q_{23} p \\ 0 \end{bmatrix}. \tag{42}$$

Since the weak equation for δw should hold for an arbitrary choice of the test vector ϕ, we are free to choose ϕ to simplify the resulting expressions. Therefore we set $\phi = \psi$, where the costate vector ψ is the solution of the adjoint equation

$$\frac{\partial \psi}{\partial t} - C_i^T \frac{\partial \psi}{\partial \xi_i} = 0 \quad \text{in } D. \tag{43}$$

At the outer boundary incoming characteristics for ψ correspond to outgoing characteristics for δw. Consequently one can choose boundary conditions for ψ such that

$$n_i \psi^T C_i \delta w = 0.$$

Then, if the coordinate transformation is such that δQ is negligible in the far field, the only remaining boundary term is

$$-\iint_{B_W} \psi^T \delta F_2 \; d\xi_1 d\xi_3.$$

Thus, by letting ψ satisfy the boundary condition,

$$Q_{21}\psi_2 + Q_{22}\psi_3 + Q_{23}\psi_4 = (p - p_d) \quad \text{on } B_W, \tag{44}$$

we find finally that

$$\delta I = -\int_{\mathcal{D}} \frac{\partial \psi^T}{\partial \xi_i} \delta Q_{ij} f_j d\mathcal{D} - \iint_{B_W} (\delta Q_{21}\psi_2 + \delta Q_{22}\psi_3 + Q_{23}\psi_4)\, p \, d\xi_1 d\xi_3. \tag{45}$$

THE NAVIER STOKES EQUATIONS

$$\frac{\partial w}{\partial t} + \frac{\partial f_i}{\partial x_i} = \frac{\partial f_{vi}}{\partial x_i} \quad \text{in } \mathcal{D}, \tag{46}$$

where the state vector w, inviscid flux vector f and viscous flux vector f_v are described respectively by

$$w = \left\{ \begin{array}{c} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{array} \right\}, \quad f_i = \left\{ \begin{array}{c} \rho u_i \\ \rho u_i u_1 + p\delta_{i1} \\ \rho u_i u_2 + p\delta_{i2} \\ \rho u_i u_3 + p\delta_{i3} \\ \rho u_i H \end{array} \right\}, \quad f_{vi} = \left\{ \begin{array}{c} 0 \\ \sigma_{ij}\delta_{j1} \\ \sigma_{ij}\delta_{j2} \\ \sigma_{ij}\delta_{j3} \\ u_j\sigma_{ij} + k\frac{\partial T}{\partial x_i} \end{array} \right\}. \tag{47}$$

In these definitions, ρ is the density, u_1, u_2, u_3 are the Cartesian velocity components, E is the total energy and δ_{ij} is the Kronecker delta function.

The pressure is determined by the equation of state

$$p = (\gamma - 1)\,\rho\left\{E - \frac{1}{2}\,(u_i u_i)\right\},$$

and the stagnation enthalpy is given by

$$H = E + \frac{p}{\rho},$$

where γ is the ratio of the specific heats. The viscous stresses may be written as

$$\sigma_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) + \lambda\delta_{ij}\frac{\partial u_k}{\partial x_k}, \tag{48}$$

where μ and λ are the first and second coefficients of viscosity. The coefficient of thermal conductivity and the temperature are computed as

$$k = \frac{c_p\mu}{Pr}, \quad T = \frac{p}{R\rho}. \tag{49}$$

It is also useful to consider a transformation to the computational coordinates (ξ_1,ξ_2,ξ_3) defined by the metrics

$$K_{ij} = \left[\frac{\partial x_i}{\partial \xi_j}\right], \quad J = \det(K), \quad K_{ij}^{-1} = \left[\frac{\partial \xi_i}{\partial x_j}\right].$$

The Navier-Stokes equations can then be written in computational space as

$$\frac{\partial (Jw)}{\partial t} + \frac{\partial (F_i - F_{vi})}{\partial \xi_i} = 0 \quad \text{in } \mathcal{D}, \tag{50}$$

where the inviscid and viscous flux contributions are now defined with respect to the computational cell faces by $F_i = S_{ij} f_j$ and $F_{vi} = S_{ij} f_{vj}$, and the quantity $S_{ij} = J K_{ij}^{-1}$ represents the projection of the ξ_i cell face along the x_j axis. In obtaining equation (50) we have made use of the property that

$$\frac{\partial S_{ij}}{\partial \xi_i} = 0 \,. \tag{51}$$

FORMULATION OF THE OPTIMAL DESIGN PROBLEM FOR THE NAVIER STOKES EQUATIONS

Suppose that the performance is measured by a cost function

$$I = \int_{\mathcal{B}} \mathcal{M}\left(w, S\right) d\mathcal{B}_{\xi} + \int_{\mathcal{D}} \mathcal{P}\left(w, S\right) d\mathcal{D}_{\xi},$$

containing both boundary and field contributions where $d\mathcal{B}_{\xi}$ and $d\mathcal{D}_{\xi}$ are the surface and volume elements in the computational domain. In general, $\mathcal{M}$ and $\mathcal{P}$ will depend on both the flow variables w and the metrics S defining the computational space. In the case of a multi-point design the flow variables may be separately calculated for several different conditions of interest.

The design problem is now treated as a control problem where the boundary shape represents the control function, which is chosen to minimize I subject to the constraints defined by the flow equations (50).

IMPLEMENTATION OF NAVIER-STOKES (EULER) DESIGN

The design procedures can be summarized as follows:

1. Solve the flow equations for ρ, u_1, u_2, u_3, p.
2. Solve the adjoint equations for ψ subject to appropriate boundary conditions.
3. Evaluate $\mathcal{G}$.
4. Project $\mathcal{G}$ into an allowable subspace that satisfies any geometric constraints.
5. Update the shape based on the direction of steepest descent.
6. Return to 1 until convergence is reached.

Practical implementation of the viscous design method relies heavily upon fast and accurate solvers for both the state (w) and co-state (ψ) systems.

SEARCH PROCEDURE

The search procedure used in this work is a simple descent method in which small steps are taken in the negative gradient direction.

$$\delta\mathcal{F} = -\lambda\mathcal{G}$$

can be regarded as simulating the time dependent process

$$\frac{d\mathcal{F}}{dt} = -\mathcal{G}$$

where λ is the time step Δt. Let A be the Hessian matrix with element

$$A_{ij} = \frac{\partial \mathcal{G}_i}{\partial \mathcal{F}_j} = \frac{\partial^2 I}{\partial \mathcal{F}_i \partial \mathcal{F}_j}.$$

Suppose that a locally minimum value of the cost function $I^* = I(\mathcal{F}^*)$ is attained when $\mathcal{F} = \mathcal{F}^*$. Then the gradient $\mathcal{G}^* = \mathcal{G}(\mathcal{F}^*)$ must be zero, while the Hessian matrix $A^* = A(\mathcal{F}^*)$ must be positive definite. Since $\mathcal{G}^*$ is zero, the cost function can be expanded as a Taylor series in the neighborhood of $\mathcal{F}^*$ with the form

$$I(\mathcal{F}) = I^* + \frac{1}{2}\left(\mathcal{F} - \mathcal{F}^*\right) A \left(\mathcal{F} - \mathcal{F}^*\right) + \ldots$$

Correspondingly,

$$\mathcal{G}(\mathcal{F}) = A\left(\mathcal{F} - \mathcal{F}^*\right) + \ldots$$

As $\mathcal{F}$ approaches $\mathcal{F}^*$, the leading terms become dominant. Then, setting $\hat{\mathcal{F}} = (\mathcal{F} - \mathcal{F}^*)$, the search process approximates

$$\frac{d\hat{\mathcal{F}}}{dt} = -A^*\hat{\mathcal{F}}.$$

Also, since A^* is positive definite it can be expanded as

$$A^* = RMR^T,$$

where M is a diagonal matrix containing the eigenvalues of A^*, and

$$RR^T = R^T R = I.$$

Setting

$$v = R^T \hat{\mathcal{F}},$$

the search process can be represented as

$$\frac{dv}{dt} = -Mv.$$

The stability region for the simple forward Euler stepping scheme is a unit circle centered at -1 on the negative real axis. Thus for stability we must choose

$$\mu_{\mathsf{max}} \Delta t = \mu_{\mathsf{max}} \lambda < 2,$$

while the asymptotic decay rate, given by the smallest eigenvalue, is proportional to

$$e^{-\mu_{\mathsf{min}} t}.$$

In order to improve the rate of convergence, one can set

$$\delta \mathcal{F} = -\lambda P G,$$

where P is a preconditioner for the search. An ideal choice is $P = {A^*}^{-1}$, so that the corresponding time dependent process reduces to

$$\frac{d\hat{\mathcal{F}}}{dt} = -\hat{\mathcal{F}},$$

for which all the eigenvalues are equal to unity, and $\hat{\mathcal{F}}$ is reduced to zero in one time step by the choice $\Delta t = 1$.

1. Quasi-Newton methods estimate A^* from the change in the gradient during the search process. This requires accurate estimates of the gradient at each time step. In order to obtain these, both the flow solution and the adjoint equation must be fully converged. Most quasi-Newton methods also require a line search in each search direction, for which the flow equations and cost function must be accurately evaluated several times. They have proven quite robust for aerodynamic optimization.
2. An alternative approach which has also proved successful in our previous work is to smooth the gradient and to replace $\mathcal{G}$ by its smoothed value $\bar{\mathcal{G}}$ in the descent process. This both acts as a preconditioner, and ensures that each new shape in the optimization sequence remains smooth.

SMOOTHING THE GRADIENT

Independent movement of the boundary mesh points could produce discontinuities in the designed shape. In order to prevent this the gradient may be smoothed. Suppose that the shape is represented in term of smooth functions such as B-splines, so that

$$\delta \mathcal{F} = Bd,$$

where d is the change of the spline coefficients. Then, using the discrete formulas, to first order the change in the cost is

$$\delta I = \mathcal{G}^T \delta \mathcal{F} = \mathcal{G}^T Bd.$$

Thus the gradient with respect to the B-spline coefficients is obtained by multiplying $\mathcal{G}$ by B^T, and a descent step is defined by setting

$$d = -\lambda B^T \mathcal{G}, \; \delta \mathcal{F} = Bd = -\lambda B B^T \mathcal{G}$$

where λ is sufficiently small and positive.

IMPLICIT SMOOTHING

Implicit smoothing may also be used. To apply smoothing in the ξ_1 direction, for example, the smoothed gradient $\bar{\mathcal{G}}$ ma be calculated from a discrete approximation to

$$\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1} \epsilon \frac{\partial}{\partial \xi_1} \bar{\mathcal{G}} = \mathcal{G}$$

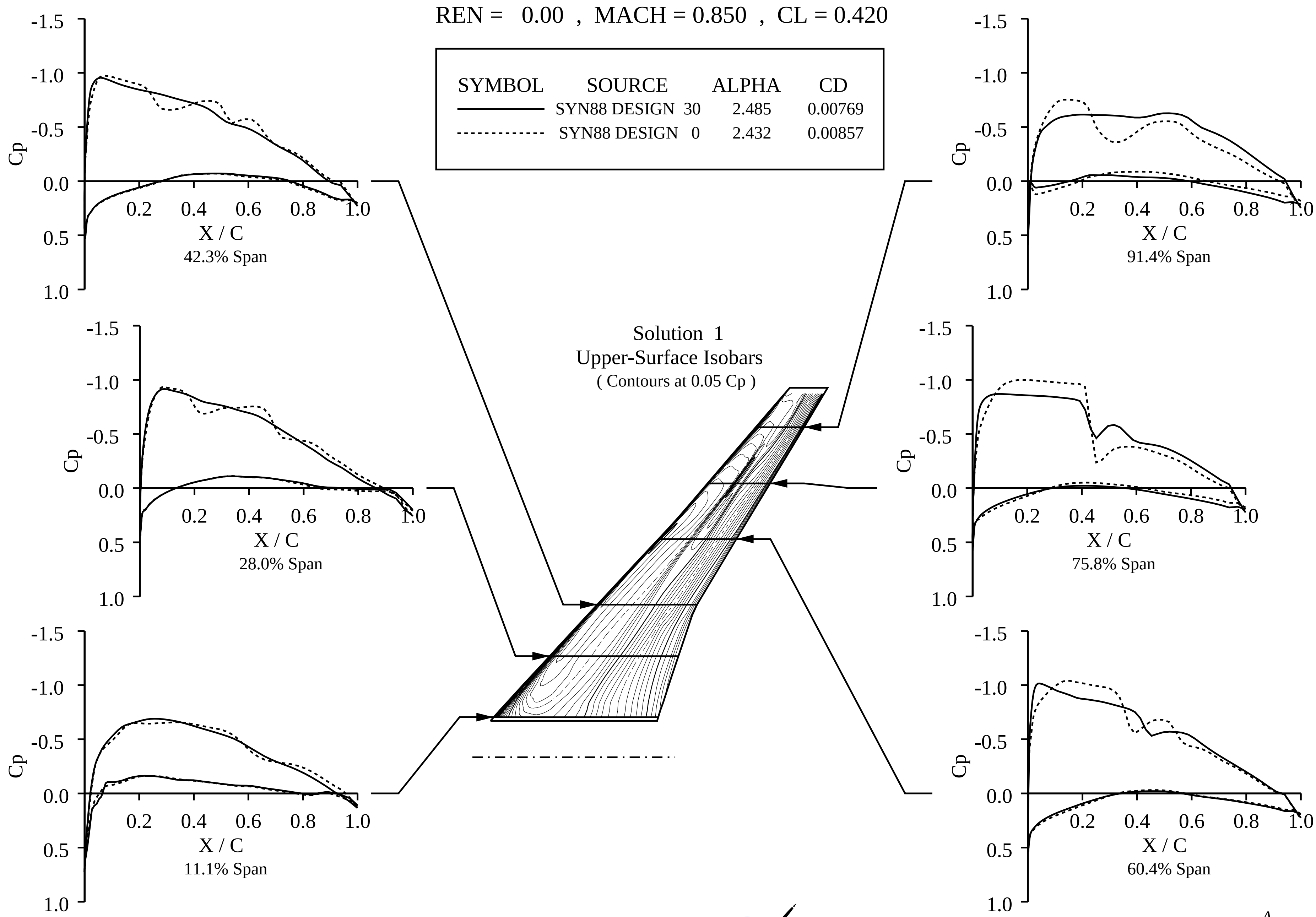
where ϵ is the smoothing parameter. If one sets $\delta\mathcal{F} = -\lambda\bar{\mathcal{G}}$, then

$$\begin{aligned} \delta I &= -\iint \mathcal{G}\delta\mathcal{F}\, d\xi_1 d\xi_3 = -\lambda \iint \left(\bar{\mathcal{G}} - \frac{\partial}{\partial \xi_1}\epsilon\frac{\partial\bar{\mathcal{G}}}{\partial \xi_1}\right)\bar{\mathcal{G}}\, d\xi_1 d\xi_3 \\ &= -\lambda \iint \left(\bar{\mathcal{G}}^2 + \epsilon\left(\frac{\partial\bar{\mathcal{G}}}{\partial \xi_1}\right)^2\right) d\xi_1 d\xi_3 < 0, \end{aligned}$$

assuring an improvement if λ is sufficiently small and positive, unless the process has already reached a stationary point at which $\mathcal{G} = 0$.

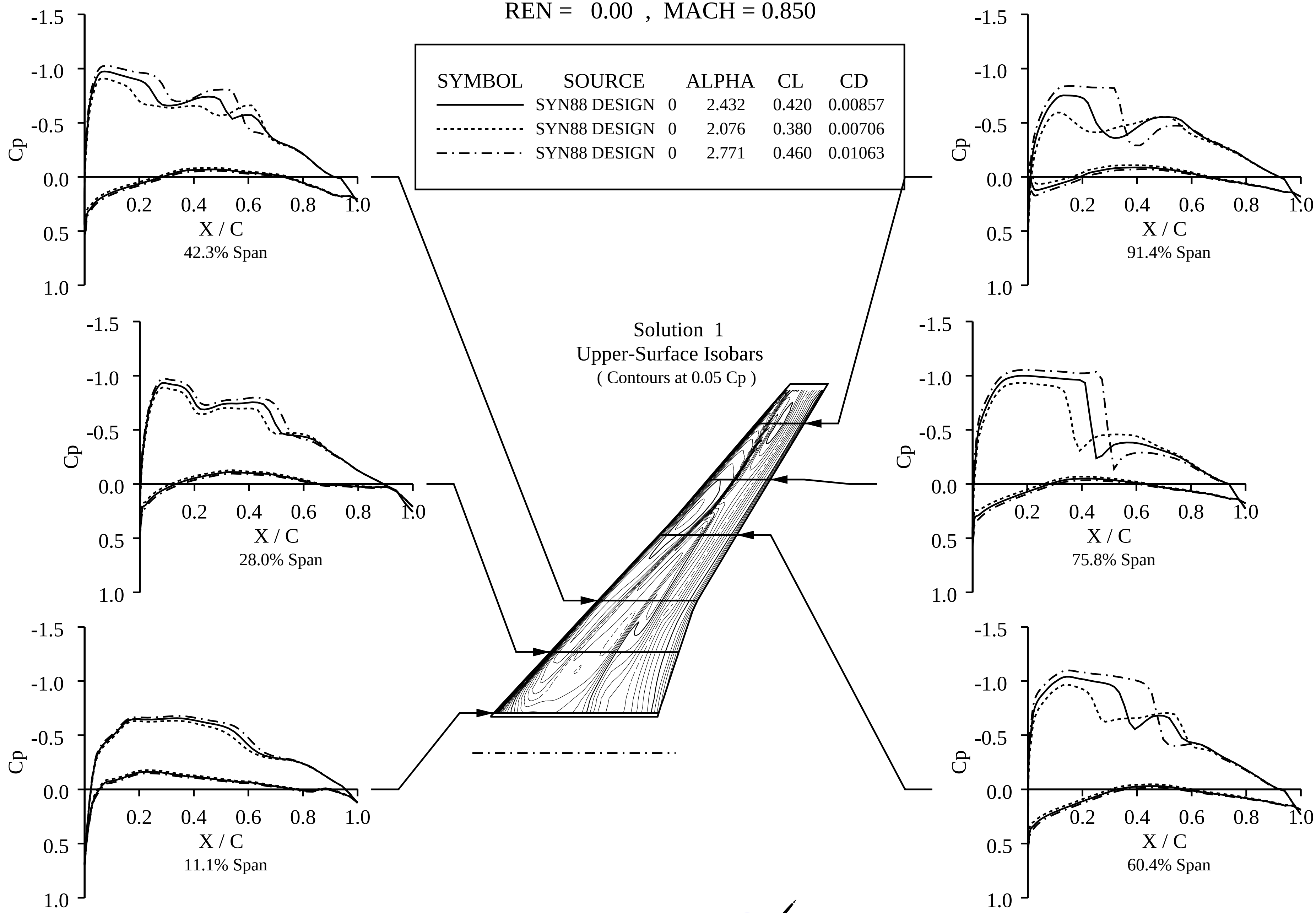
$\boxed{\text{This results in very large savings in the computational cost.}}$

COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
BOEING 747 WING-BODY



COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
BOEING 747 WING-BODY

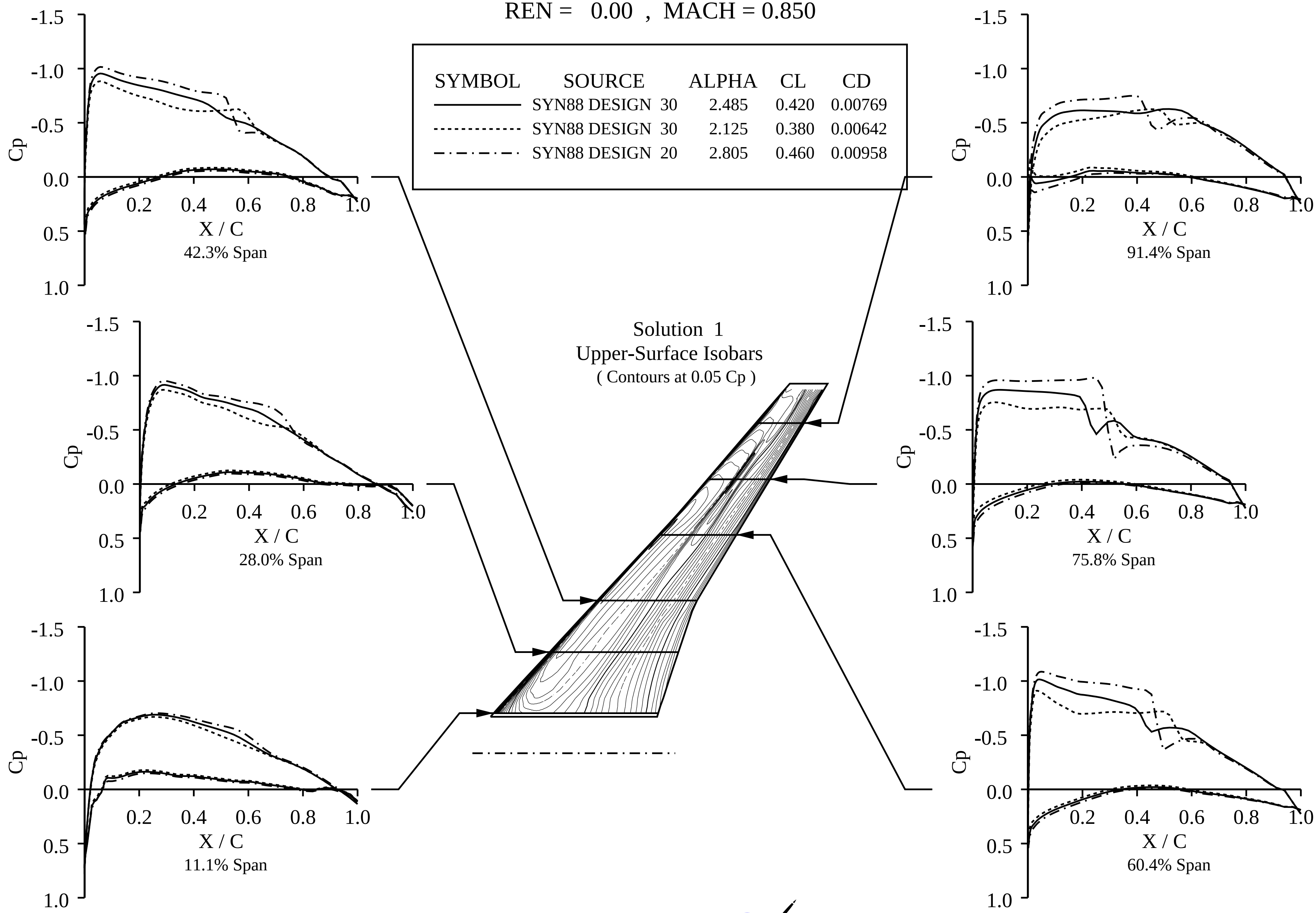
REN = 0.00 , MACH = 0.850

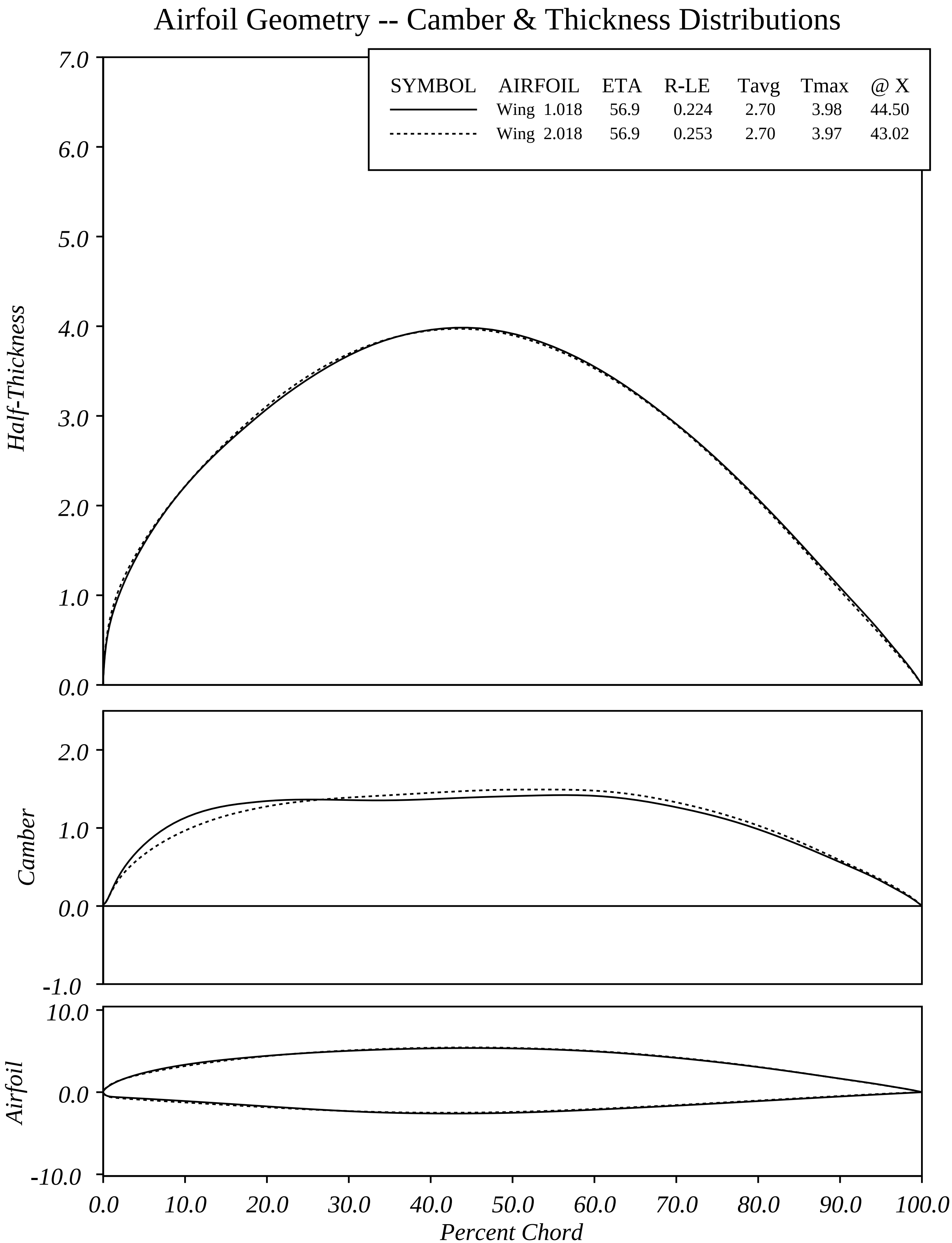


COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
BOEING 747 WING-BODY

REN = 0.00 , MACH = 0.850

SYMBOL	SOURCE	ALPHA	CL	CD
——— (solid)	SYN88 DESIGN 30	2.485	0.420	0.00769
- - - - (dotted)	SYN88 DESIGN 30	2.125	0.380	0.00642
— · — · (dash-dot)	SYN88 DESIGN 20	2.805	0.460	0.00958

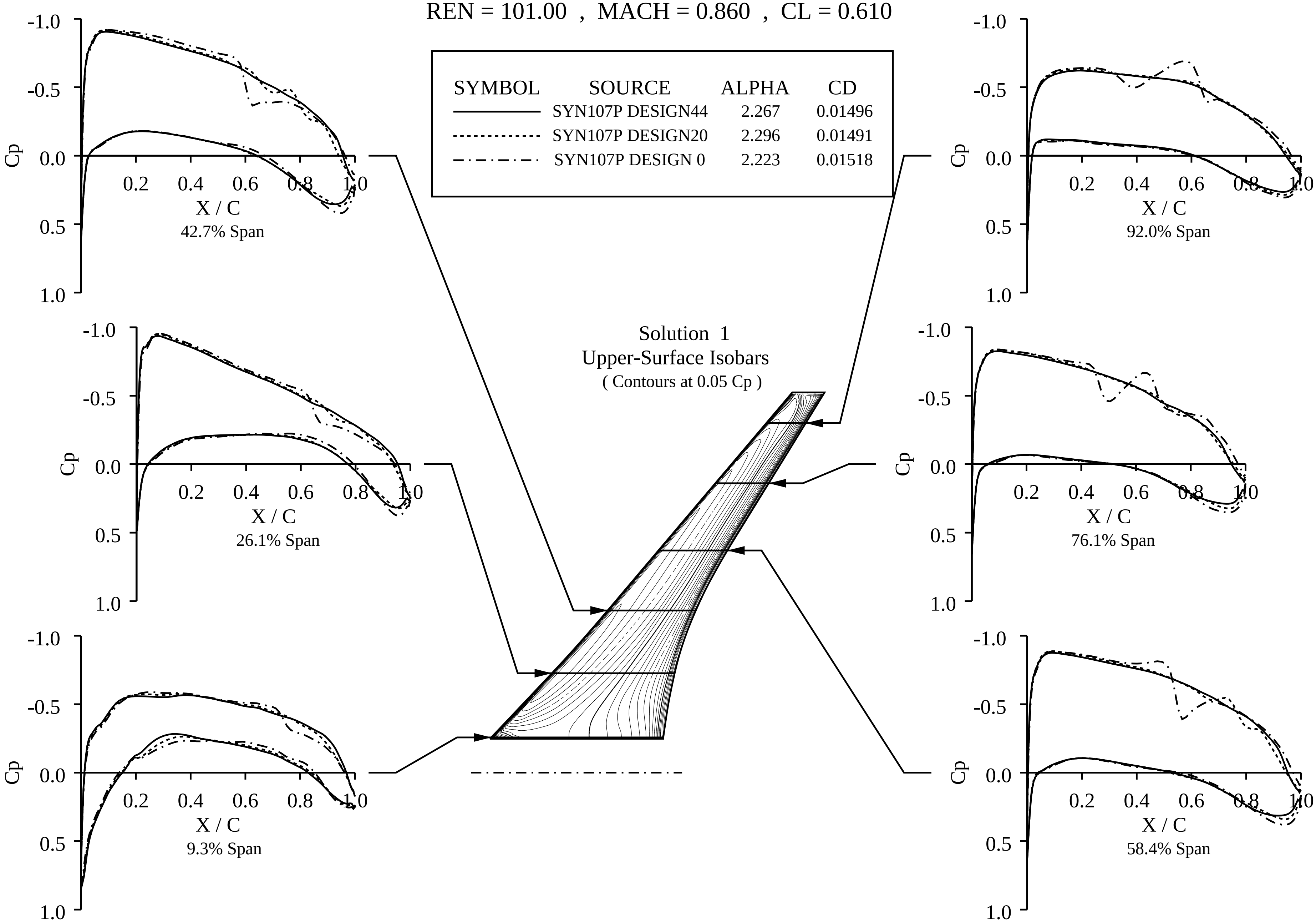




Airfoil Geometry -- Camber & Thickness Distributions
SYMBOL AIRFOIL ETA R-LE Tavg Tmax @ X
Wing 1.018 56.9 0.224 2.70 3.98 44.50
Wing 2.018 56.9 0.253 2.70 3.97 43.02
Half-Thickness
Camber
Airfoil
Percent Chord
0.0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0

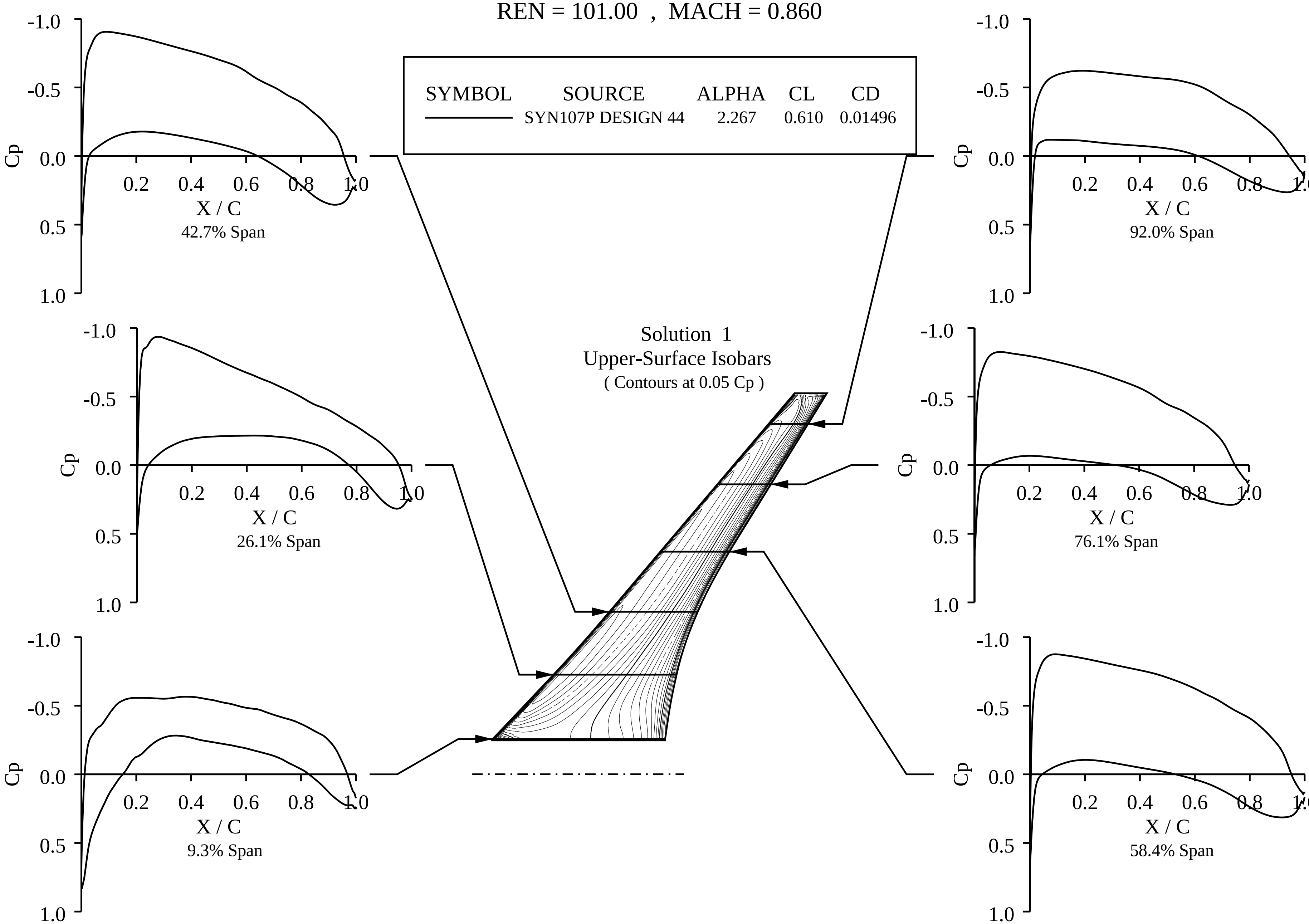
COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
MPX5X WING-BODY

REN = 101.00 , MACH = 0.860 , CL = 0.610



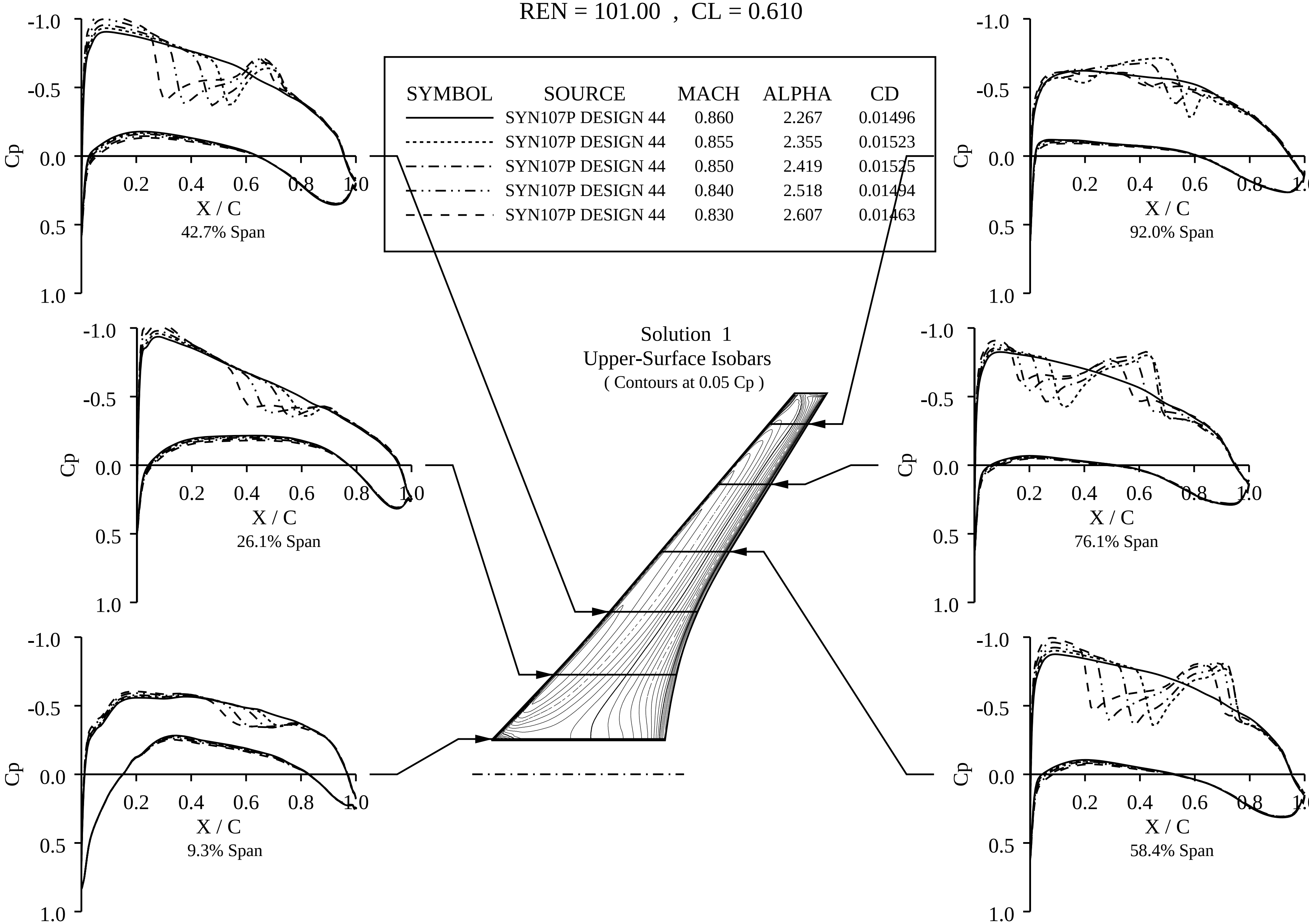
COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
MPX5X WING-BODY

REN = 101.00 , MACH = 0.860



COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
MPX5X WING-BODY

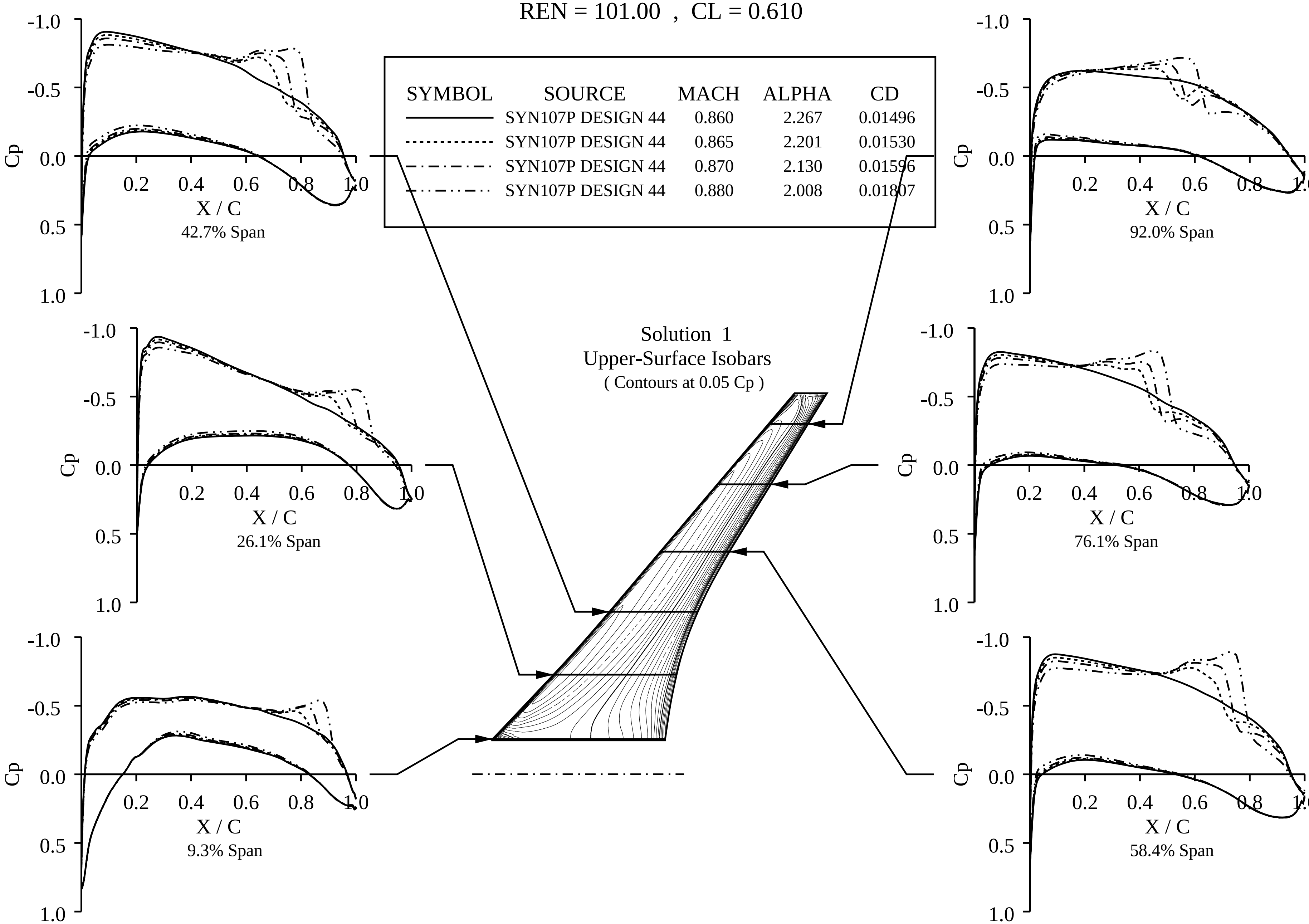
REN = 101.00 , CL = 0.610



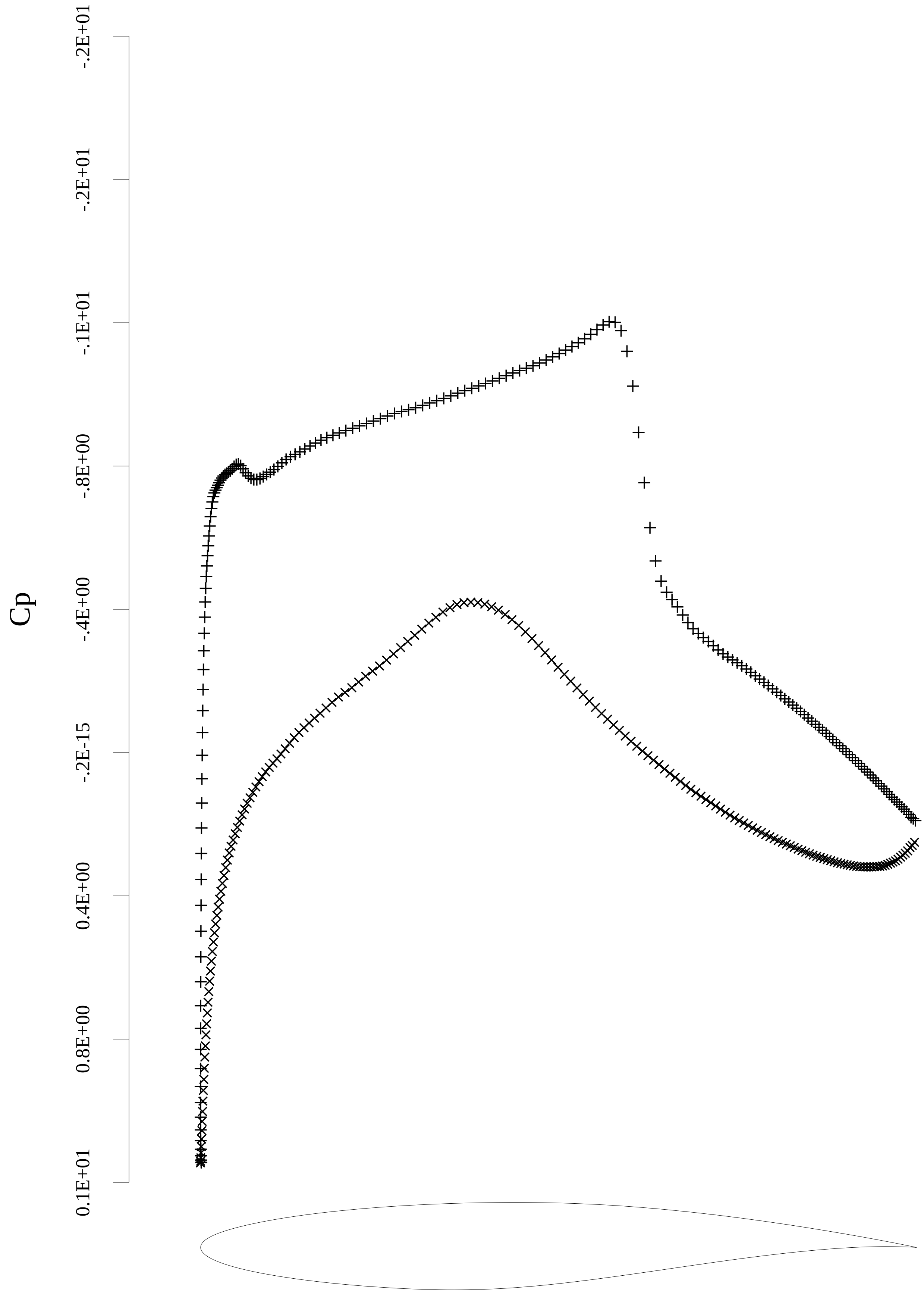
COMPPLOT
JCV 1.13

COMPARISON OF CHORDWISE PRESSURE DISTRIBUTIONS
MPX5X WING-BODY

REN = 101.00 , CL = 0.610



COMPPLOT
JCV 1.13

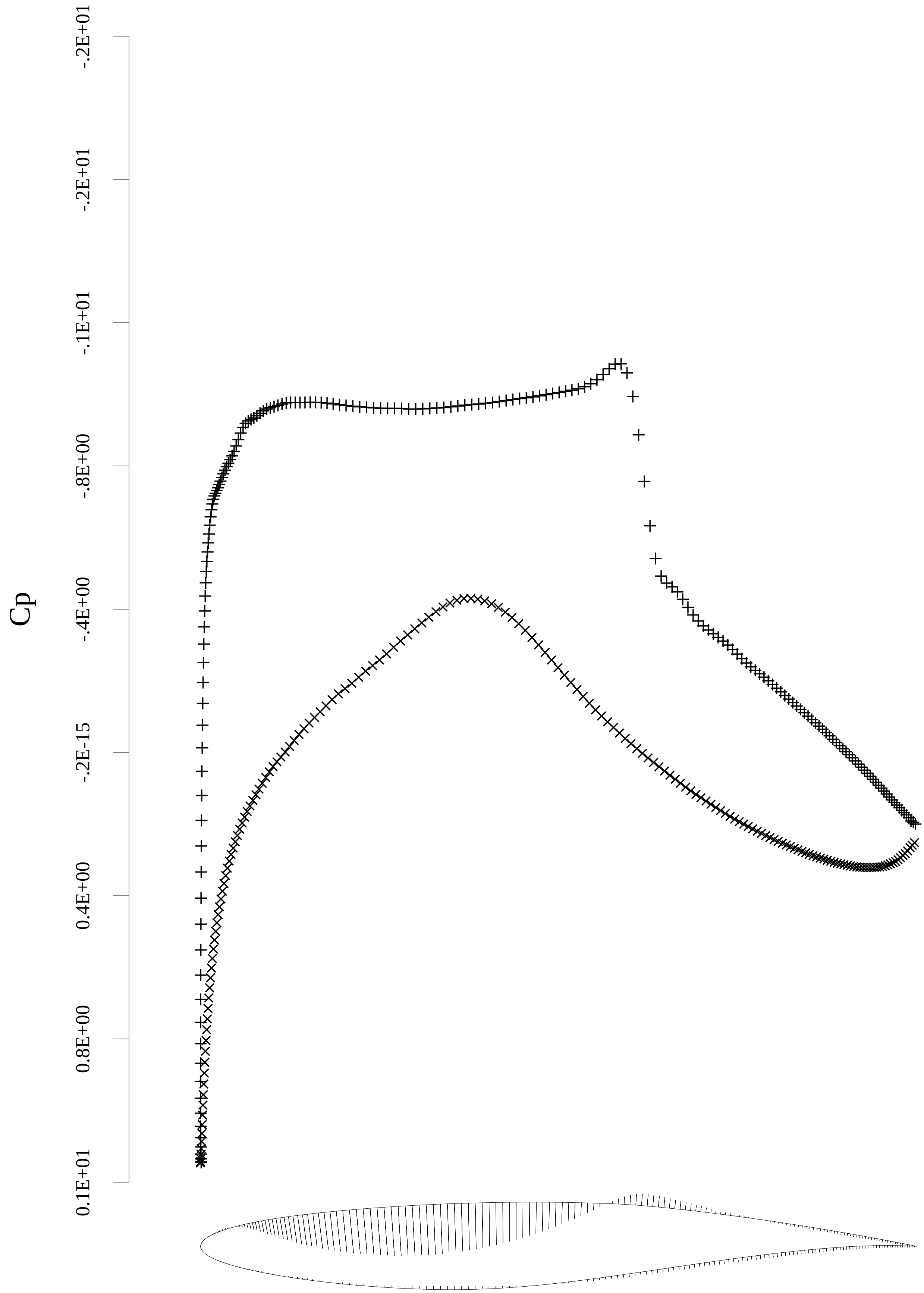


RAE 2822 DRAG REDUCTION

MACH 0.750 ALPHA 1.632

CL 0.6449 CD 0.0091 CM -0.1054 CLV 0.0001 CDV 0.0056

GRID 512X64 NDES 0 RES0.379E-01 GMAX 0.100E-05



RAE 2822 DRAG REDUCTION

MACH 0.750 ALPHA 1.653

CL 0.6510 CD 0.0054 CM -0.0993 CLV 0.0000 CDV 0.0057

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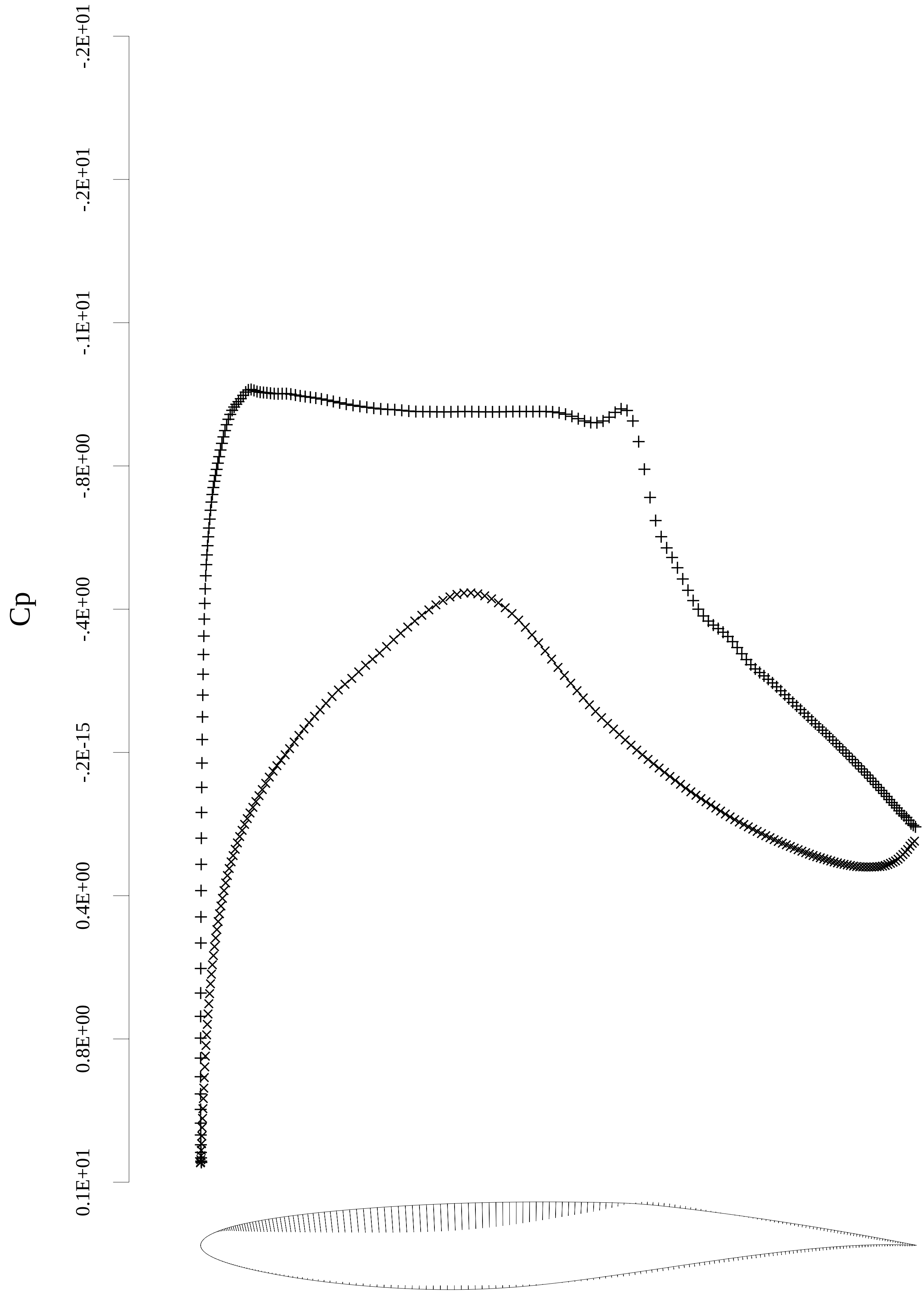
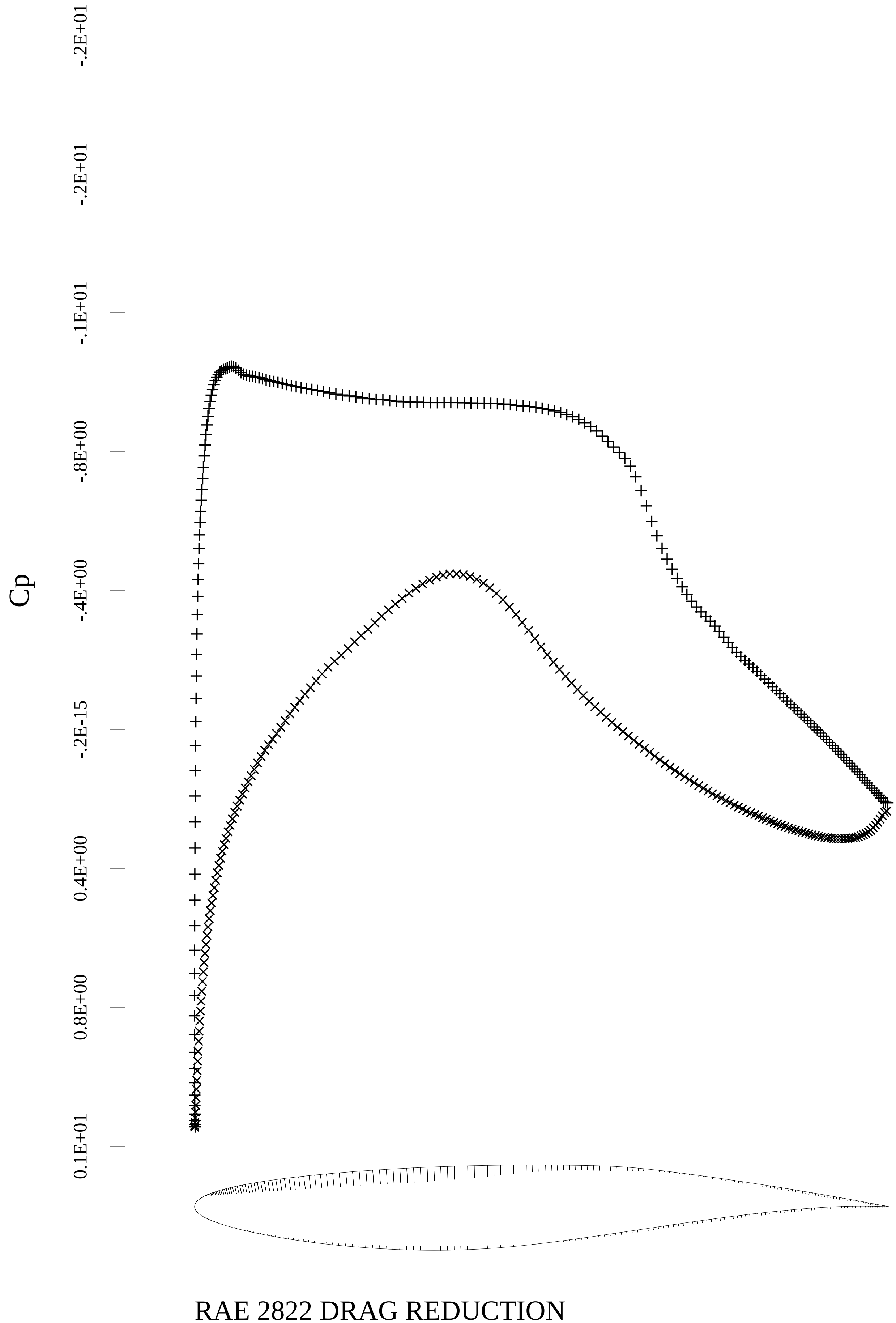


RAE 2822 DRAG REDUCTION

MACH 0.750 ALPHA 1.695

CL 0.6478 CD 0.0041 CM -0.0954 CLV 0.0000 CDV 0.0058

GRID 512X64 NDES 5 RES0.427E-01 GMAX 0.318E-02

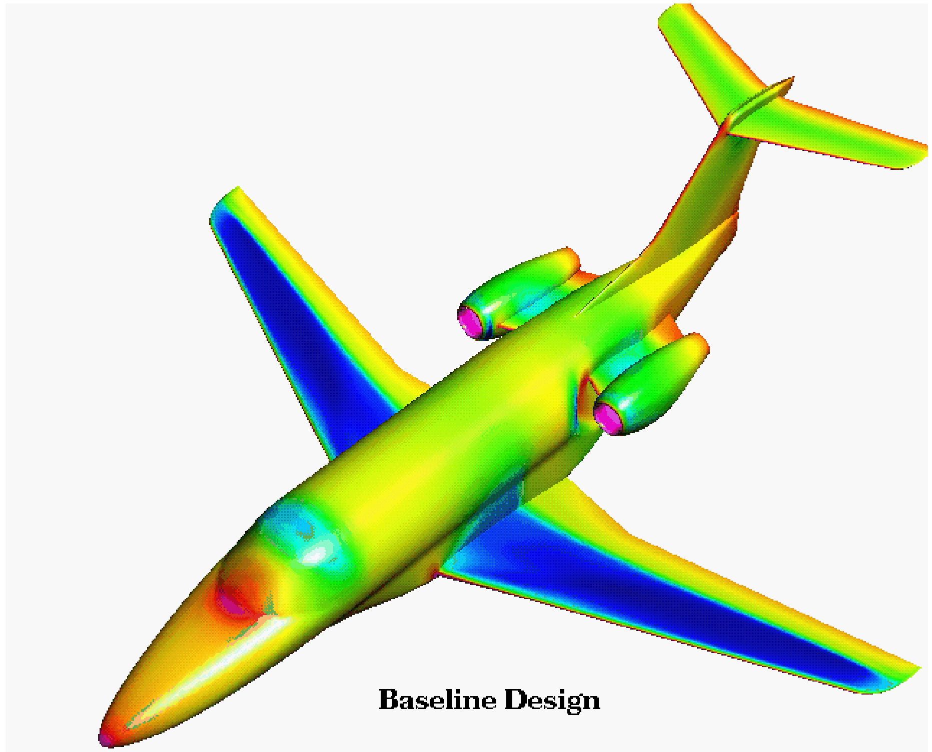


RAE 2822 DRAG REDUCTION

MACH 0.750 ALPHA 1.813

CL 0.6460 CD 0.0041 CM -0.0910 CLV 0.0000 CDV 0.0058

GRID 512X64 NDES 10 RES0.220E-01 GMAX 0.178E-02



Baseline Design