Abstract

Future air traffic management (ATM) decision support tools (DST) rely upon trajectory forecasting to adequately deliver benefits. These forecasts are subject to errors from a variety of sources. As the number and sophistication of ATM DST capabilities grow, the interoperability between DST will become more sensitive to the magnitude and variation in trajectory forecasting errors across DST. This paper presents a parametric analysis of some of the larger error sources. Errors are considered due to turn dynamics, weight, wind gradient omission, speed intent, interim altitudes, top of descent placement, and wind for typical airborne flights. Estimates of error contributions to typical scenarios are presented as a function of phase of flight. While certain errors are isolated as having larger impacts on the typical scenario, all error sources considered may have significant impact on trajectory forecasting under specific circumstances.

1 Introduction

In the quest to modernize our national airspace system (NAS) to reduce congestion and delays, the Federal Aviation Administration must develop, deploy, and maintain new Decision Support Tool (DST) automation. The goal of such automation is to help controllers manage greater levels of traffic safely, efficiently, and with greater productivity. The FAA, with assistance from the National Aeronautics and Space Administration (NASA), has been successful in deploying the first phase of “free flight” DSTs to a subset of our nation’s Air Traffic Control (ATC) facilities. Although each of these tools provides a valuable benefit to a unique region of airspace (e.g., terminal, en route) and type of operations (i.e., local control, local TFM, national TFM, collaborative decision making), they all share one aspect in common, trajectory modeling. Each tool, in one form or another, must generate its advisories based on the prediction and analysis of four-dimensional (4D) trajectories for each flight operating within its airspace domain.

Although each new FAA DST must generate a return on investment for deployment and maintenance, an issue arises with the large-scale success of many DST capabilities. Each tool developed to date has also had to develop its own trajectory modeling capability. This independent development approach leads to overlapping efforts with some duplication across DST projects. Furthermore, differences in requirements and approach, when combined with a lack of standardization, lead to subtle and sometimes major differences in software and architectural implementation of trajectory modeling functions. As a result of these differences, two issues arise regarding the precision (i.e., interoperability) across DST, and the cost of development, deployment, and maintenance of a diverse set of systems performing a similar function.

First, differences across DSTs could potentially lead to situations where controllers and/or traffic managers receive slightly different values for the same parameter for the same flight. Such differences may be due to differences in the level of modeling fidelity, input data, and update rates. Setting aside the issue of trajectory prediction accuracy, this issue is one of precision between different DST. For example, a conflict-
probe tool may indicate that a flight will arrive at a fix at one time, and a separate flow-metering DST may indicate a slightly different time. In many cases, such differences may be entirely innocuous. However, such differences may lead to interoperability differences that are unacceptable for more advanced applications.

The second issue has primarily to do with the operation and maintenance of an operational DST. In many cases, the initial innovation represents only a fraction of the total life-cycle cost of an advanced automation system. When one considers that each DST is typically deployed to many ATC facilities (up to 20 en route Centers and many more terminal approach controls), it becomes clear how the cost of developing and deploying a trajectory modeling function for each tool is magnified by the number of installations (throughout the nation) that must be installed, monitored, and maintained. Differences in trajectory modelers require duplicative efforts to develop, parallel efforts to train facility-support personnel, and a greater number of support personnel to service the diverse set of DST systems.

Common trajectory modeling (TJM) refers to a capability to provide common “services” to subscribing DST. Prior efforts [1,2] provide a detailed description of the potential levels of common TJM services. They also define a research and development approach for assessing and down-selecting the type of common TJM services and their implementation. Many prior efforts [3,4,5] have identified the impact of modeling and input errors on performance of specific trajectory modelers or decision support tools. However, one must recognize that these input errors are not static. For example, future information exchange between the airspace users and ATSP may improve the ATSP’s knowledge of aircraft weight for estimating climb and descent profiles. As the magnitude and frequency of these input errors evolve, so does the relative importance of each error source. This paper presents a parametric analysis of some of these error sources allowing a future user to estimate the relative contribution due to each error source. We also apply estimates of these errors to a scenario day to provide an assessment of error contributions under today’s operational environment.

2 Trajectory Forecasting Errors

As part of the development of a research plan for investigating common trajectory modeling, a joint FAA-NASA-MITRE effort catalogued the specific modeling “factors” contributing to modeling errors. Each factor represented an element of a model (e.g., inclusion of turn dynamics), or an input error (e.g., improper route amendments, uncertainty in winds). These factors were subsequently categorized qualitatively according to the group’s expectation of error magnitude (high-medium-low), frequency of error (frequent or rare), and time horizon within which the FAA could develop mechanisms for coping with the uncertainty. For example, the inclusion of turn dynamics could be accomplished within TJM functions with relative ease, but improvements in pilot intent require the ability to obtain the information from the flight deck to a trajectory modeler. Identified errors are described in [6].

In this paper, we focus on some of the higher impact errors (either high due to impact or frequency) that can be quantitatively analyzed. Specifically, we discuss and estimate the impact of the following factors:

- Inclusion of turn dynamics in the trajectory modeler.
- Error in aircraft weight estimate.
- Omission of wind gradient term in modeler.
- Error in speed intent.
- Interim altitude levels on climb and descent.
- Error in placement of top of descent.
- Error in estimates of wind.
3 Results

Two types of results are presented in this section: parametric analyses, and averaged results across a scenario day. The parametric analyses are sensitivity studies intended to illustrate the impact of a single factor on trajectory uncertainty as the impact level of that factor is increased. For example, the impact of weight on accuracy during climb is reported as the weight error is increased. Averaged results across a scenario day represent the standard errors that are expected in trajectory forecasting when subject to errors on input representative of today’s operations. The averaged results include the effect of the error on a collection of flights subject to a variety of different conditions and parameters (e.g., winds, aircraft type, flight profile, cruise level, initial weight).

This report presents the trajectory uncertainty during the en route portion of flight (defined here as that portion above 10,000 feet). We further subdivide the en route portion into three distinct phases: climb, cruise and descent.

Trajectory uncertainty is defined in terms of three separate components: along-track, altitude and cross-track. Errors are obtained by defining a vector between the “truth” trajectory and the predicted trajectory. This vector is projected into a component in the vertical direction (altitude error), an along-track component in the direction of the “truth” velocity vector, and a cross-track component in the horizontal plane normal to the truth velocity vector.

3.1 Turns

Certain trajectory forecasting tools assume instantaneous turns rather than modeling the dynamics of the turn. We estimate the impact of this omission by comparing trajectories with a simple turn model to trajectories assuming a discrete heading change at a waypoint. This error represents the error of exclusion of the turn model and does not attempt to represent errors introduced in the execution of the turn by a pilot (described in [5]). Our parametric analysis assumes a turn model, representing truth, with constant airspeed, a maximum bank angle of 25 degrees, no winds and the bank angle is achieved instantaneously.

Figure 1 shows the turn model employed. The along-track error and maximum cross-track errors are respectively given by:

\[
2R \tan\left(\frac{\Delta \psi}{2}\right) - R \frac{\Delta \psi}{2}
\]

(1)

\[
R \left( \frac{1}{\cos^2\left(\frac{\Delta \psi}{2}\right)} - 1 \right)
\]

(2)

Figure 2 illustrates the effect of airspeed and heading change on the maximum cross-track and along track error in a turn. (e.g., An error of 8557 ft. along track and 9270 ft. across track can be experienced by a flight at 400 kts encountering an 80-degree heading change.)

A different turn model was used to obtain the impact of turns on a scenario. This turn model incorporated the effect of winds on the turn and assumed a standard rate turn with a bank angle limited to a maximum of 25 degrees. The wind was assumed to be a constant during the turn. An entire day’s worth of flight plans and actual tracks were obtained, and the errors introduced by modeling instantaneous turns were investigated. Figure 3 shows the distribution of maximum cross-track errors for both types of scenarios. The average maximum cross-track error (averaged across all turns) is 1711 feet. Figure 4 illustrates the along-track standard error during all en route phases of flight based upon actual turn data. Note that turns in a terminal environment would likely be larger and more frequent.

3.2 Weight Error

We estimated the impact of a percentage error in aircraft weight during both climb and descent. For the parametric analysis, we assume an aircraft initially accelerating from 250 knots to a climb calibrated airspeed (CAS) at a climb rate of 2000 feet per minute, the aircraft then climbs...
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using a constant CAS and constant Mach number segment. Upon descent, the process is reversed.

As an illustration of the impact of weight on the climb angle ($\gamma$), an aircraft with a generic drag polar ($C_{D0} + kC_L^2$) subject to no wind would experience the following climb angle.

$$\sin(\gamma) = \left(1 + \frac{1}{g \frac{dV}{dh} \frac{dt}{dh}}\right)^{-1} \left(\frac{T - \frac{1}{2} \rho V^2 \frac{d}{dh} \frac{dV}{dh}}{W} - \frac{kW}{\left(\frac{1}{2} \rho V^2 s\right)^2} \right)$$

(The $dV/dh$ term is obtained by holding either Mach or CAS constant). During climb, the thrust ($T$) term dominates over the drag terms and we see that the climb rate would be dominated by a term inversely proportional to the weight. During descent (negative $\gamma$), the impact of the weight on descent rate will depend on whether the induced drag term ($kW$) dominates. Thus, for certain aircraft, weight errors will yield almost no error in altitude on descent (e.g., see [4]).

We applied variations in weight of 5 and 10% above and below a nominal weight of 100000 lbs to a B737-300 aircraft model climbing to 33000 feet. No wind was used for the parametric analysis, but an ISA atmosphere was assumed for the temperatures. Figures 5 and 6 illustrate the altitude and along-track errors experienced by the flight during the climb. During the Mach segment, true airspeed decreases as altitude increases. The reverse is true during the CAS segment. Thus, the along-track error tends to peak around transition.

Using a sample of flights across the NAS, we calculated the climb and descent profiles at nominal weights, and compared them to climb profiles with an additional weight uncertainty expressed as a percentage error. This error was drawn from a normal distribution with zero mean and variance of 5% to be consistent with [7]. (Assuming that weight biases in [7] could be removed.) For descent profiles, we assumed the same percentage weight error. Figure 7 shows the standard error encountered both along track and in altitude during climb. The altitude error peaks at 763 feet decreasing to zero as the cruise altitude is reached. The along track error grows to a constant 0.41 nautical miles.

### 3.3 Wind Gradient Omission

Many tools seeking to forecast trajectories include the effect of winds on climb and descent, but neglect the effect of the wind gradient term. This term is best illustrated by an example. Consider a flight subject to a tailwind ($w$) monotonically increasing as a function of altitude. The climb angle ($\gamma$) can be expressed as:

$$\sin(\gamma) = \left(\frac{T - D}{m} \left(\frac{dV}{dh} + \frac{dw}{dh}\right) + g\right)^{-1}$$

An increase in the wind gradient ($dw/dh$) will result in a decrease in the climb rate. This result can be observed in Figures 8 and 9. Linear wind gradients were imposed assuming wind speeds of up to 100 knots at 33000 feet. In these figures, a positive number implies a tailwind. The lack of symmetry in the along-track error is the result of competing effects. When subject to a headwind, neglecting the gradient places the estimated flight below the actual flight. This results in a lower headwind at the lower altitude (increasing the ground speed). The lower altitude also results in a lower true airspeed for constant CAS. However, as the flight climbs into the constant Mach regime, the lower altitude results in a higher airspeed. At lower altitudes, the effect of constant CAS dominates, thus the estimated flight lags behind the actual. At higher altitude, the wind effect dominates over the Mach number effect. The estimated flight then begins to catch up. When subject to a tailwind, the dominant effect always results in a faster estimated flight.
3.4 Speed Intent

Constant speed intent errors do not lead to a bounded error. These errors integrate into increasing positional uncertainty. We investigate the impact of speed uncertainty in climb and descent by applying a constant percentage error in speed to the climb and descent CAS and Mach. From [7] we estimate a speed error of approximately +/-5% in both climb CAS and Mach number. This assumes an error stemming from the observed variance in pilot-discretionary speed profile for cases where the DST is attempting to guess a flight’s planned speed versus providing a speed advisory. On descent, an error of 5.5% was imposed. During descent, the along-track error does not continue to increase since all flight trajectories are bound by the 250 kt (CAS) restriction below 10,000 feet.

When applied to the climb scenario, the along-track standard error grows at 5.3 knots. During descent, the error peaks at 3.1 nmi. Altitude errors peak at 212 feet in climb and 2920 feet in descent.

3.5 Interim Altitudes

One additional source of trajectory uncertainty involves the practice of aircraft flying level at altitudes other than their cruise altitude. This can occur for a variety of reasons:

- transitioning aircraft will encounter a conflict that is averted by remaining at an interim altitude,
- an interim altitude is provided to a transitioning flight and the controller does not provide a continuing clearance until the aircraft has leveled off
- a procedure is in place with a specified altitude restriction
- a flight conducts a “step-climb” to a new cruise-level
- a flight requests a new altitude to avoid turbulence.

We estimated the frequency with which flights encounter a “level-off” altitude during climb by looking at a sample of 3966 climbing flights in a scenario day. A total of 28% of these climbing flights were subject to interim altitudes with a level-off altitude distribution, and duration as shown in Figure 10. Note that longer durations tended to be associated with level-offs at higher altitudes due to step climbs.

The impact of these level-offs on trajectory uncertainty was estimated by assuming that none of the level-offs were predicted to occur when the trajectory forecast was made. (Although, some of these, based on restrictions, would be known.) Figure 11 illustrates the impact of these level-offs on climb. During climb, level-offs will always place the flight path below forecast, resulting in the average peak altitude error of 280 feet. At one sigma, the peak altitude error is 1418 feet.

The above approach assumes that interim altitudes are not known prior to conducting trajectory forecasts. However [8] found that when dealing with the input of altitude clearances into automation, “almost all altitude clearances were correlated to a flight plan amendment or interim altitude message”. Thus, at some point prior to the level-off, the altitude of the level-off would be known to the automation either through amendments, or through restrictions. We analyzed the errors by assuming that the level-off altitude was known, but the duration was assumed to be the mean. In this case, knowing the level-off altitude lowers the peak altitude error from about 1400 feet to 900 feet.

3.6 Top of Descent

One error occurring only in descent involves the placement of the top of descent (TOD) point. A delay in the placement of top of descent will result in a flight significantly above the forecasted profile. Some trajectory prediction algorithms will integrate backwards from bottom of descent, for these algorithms, one can approximate the error as a scaled error in the placement of the bottom of descent.
We estimated the error in TOD placement by first looking at the variance in the top-of-descent based upon observed flight data. We calculated the variance by grouping flights by arrival airports and corner posts. This approach sought to eliminate regional variation in TOD placement. Based upon these observations, we found a variance of 28 nautical miles in the TOD placement. We estimated this error by imposing a normally distributed error in TOD with a variance of 28 nautical miles.

Note that an error of 28 nautical miles on TOD represents the error encountered under current operations by a decision support system without imposing changes in descent procedures. When these assumptions are relaxed, [9] showed that a mean error of 1.2 nautical miles could be obtained in TOD placement for non-FMS equipped aircraft (assuming a TOD advisory is issued). Given accurate wind, performance, and speed-profile data, [9] showed the error could be reduced to 2.4 nautical miles for FMS aircraft without requiring advisories.

3.7 Wind

Unlike some of the errors reported previously, errors in forecasting the wind can be represented by a vector field, rather than a point value. However, much of the literature reports wind prediction accuracy in terms of aggregate rms statistics (see [10] for a discussion on these problems) which hides localized wind errors that are on a scale of relevance to conflict probe applications (e.g. 20 mins). Furthermore, the impact of wind errors on trajectory uncertainty depends on the spectral characteristics of the wind estimation error. Wind uncertainty also depends on the forecast being used (e.g., RUC-1, RUC-2)[11] or if the forecast was augmented with updates from airborne data (e.g., [12]).

We present here a parametric analysis of wind uncertainty in climb and in descent, assuming a wind bias error corresponding to rms values typically reported in the literature. A bias error is used since one would expect errors resulting from time varying wind signals to have a smaller impact on trajectories than the effect of a constant wind bias. The magnitude of the wind bias was obtained from [10,11,12] with rms values ranging from 7 to 20 knots.

Along-track error grows monotonically with the speed error. The altitude error is zero when expressed as a function of time since the aircraft continues to climb at the same rate in an air-fixed frame of reference. Note that if altitude restrictions are in place, a coupling between along-track and altitude will result in altitude errors due to along-track errors.

We obtained the impact of the wind error on a scenario day by applying a wind error to each flight in our climb and descent scenarios. The wind error was obtained by sampling from a distribution using the cumulative probability density function shown in [10]. This wind bias produced along-track errors growing at 10 kts.

3.8 Lateral Deviations not Amended

One effect that we did not calculate in this report, is the impact of route deviations for which amendments were not entered into the automation system. Through a voice tape analysis, [8] determined that only 18% of route clearances are entered into the automation as route amendment messages. While this number is based on a limited data set, the impact of errors in the lateral route were investigated in [3,7]. The former study revealed a mean error of 2.46, 1.09, and 6.06 nautical miles for departures, over-flights and arrivals. Standard deviations for the same scenarios were 2.32, 1.48 and 5.51 nautical miles, respectively.

4 Summary

We summarize the results of the parametric analysis in Table 1. For each parametric analysis, we obtained an error as a function of forecast time (e.g., Figures 5,6,8,9). Table 1 reports the peak error occurring across all time. For the turns, the cross-track error is reported in column 4. For some of the factors (e.g., wind bias), the along-track error continues to grow at
a particular rate and does not peak. For these errors, the growth rate in the error is reported in knots. The level-off error depends on both the magnitude of the level-off and the altitude of the interim altitude. For this table, we considered interim altitudes at 15,000 feet.

Table 1. Parametric analysis, peak errors.

<table>
<thead>
<tr>
<th>Factor</th>
<th>Input Parameter</th>
<th>Along-track or Alt. (ft)</th>
<th>Phase*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turns</td>
<td>20°</td>
<td>0.02, 592</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>0.27, 3165</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>90°</td>
<td>2.71, 15912</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>-0.92, -2206</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>5%</td>
<td>-0.46, -1137</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>0.44, 1183</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>0.39, 2385</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>1.05, 1104</td>
<td>(2)</td>
</tr>
<tr>
<td>Weight</td>
<td>5%</td>
<td>0.57, 594</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>-5%</td>
<td>-0.62, -661</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>-1.29, -1372</td>
<td>(3)</td>
</tr>
<tr>
<td>Wind</td>
<td>100 kts</td>
<td>0.79, 1253</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>50 kts</td>
<td>0.31, 649</td>
<td></td>
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<tr>
<td></td>
<td>-50 kts</td>
<td>-0.18, -698</td>
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<tr>
<td></td>
<td>-100 kts</td>
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<td>(3)</td>
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<td></td>
<td>100 kts</td>
<td>-2.40, -1804</td>
<td>(3)</td>
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<td></td>
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<td></td>
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<tr>
<td></td>
<td>-100 kts</td>
<td>0.72, 1927</td>
<td></td>
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<tr>
<td>Speed</td>
<td>5%</td>
<td>16 kt, 595</td>
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<td></td>
<td>5%</td>
<td>17 kt, -1061</td>
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<tr>
<td></td>
<td>5.5%</td>
<td>2.54, -5558</td>
<td>(3)</td>
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<tr>
<td>TOD</td>
<td>25 nmi</td>
<td>9.5, 9660</td>
<td>(3)</td>
</tr>
<tr>
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<td>5 nmi</td>
<td>1.9, 2213</td>
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<td></td>
<td>-5 nmi</td>
<td>-1.9, -2215</td>
<td></td>
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<tr>
<td></td>
<td>-25 nmi</td>
<td>-9.5, -9620</td>
<td></td>
</tr>
<tr>
<td>Level-off</td>
<td>w kts</td>
<td>w kt</td>
<td>-</td>
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<tr>
<td>Wind</td>
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<td>1.3, 2180</td>
<td>(3)</td>
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<tr>
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<td>(3)</td>
</tr>
<tr>
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<td>1.5, 3352</td>
<td>(1)</td>
</tr>
<tr>
<td></td>
<td>5 min</td>
<td>5.3, 10310</td>
<td></td>
</tr>
</tbody>
</table>

Table 2 summarizes the results of the application of errors to our scenarios (e.g., Figures 4, 7). Each error in the table represents the peak in the standard error time series. In certain cases for which the mean was sufficiently large, we also report the mean in brackets.

On descent, improvements in top-of-descent, wind uncertainty, speed intent and altitude intent data will provide the largest reduction (of the factors considered) in typical altitude and along-track errors. During climb, reductions in altitude intent and weight uncertainty will provide the best improvements in altitude error, whereas, better wind prediction and speed intent will provide the best reduction in along-track error.

Depending on the DST, consideration of the average errors may yield unacceptable trajectory forecasting errors. For conflict-probe...
applications, Table 1 reveals that almost all factors have certain circumstances under which the errors may be considered large compared to the 5 nm lateral and 1000 ft vertical separation requirements.

For applications involving turn advisories, turns may have a significant cumulative effect on along-track error. Neglecting turn dynamics may provide palatable errors on average, but for a flight with large turns for metering and spacing, the error would become unacceptable.

While the effect of the wind gradient did not figure prominently in the average, large wind gradients would affect all flights in a geographical location. Thus, a large wind gradient over a hub airport would contribute significantly to trajectory forecast errors to almost all climbing and descending flights into and out of that hub.

Some errors can be addressed by improving the flow of information between humans and automation, or by defining procedures. An example is provided in [9] whereby procedures and/or data can be used to significantly reduce the top of descent uncertainty from the large errors reported herein to the value shown as the second TOD row of Table 2.

5 Conclusions

As future technologies deploy within the NAS, information currently unavailable will become more readily obtainable by trajectory forecasters. Thus the magnitude of the errors reported in Table 2 will evolve as the NAS modernizes. Furthermore, trajectory forecasting requirements will depend on the DST application. By analyzing future DST for trajectory forecasting requirements as a function of the anticipated level of fidelity of all the input factors, requirements for a common trajectory modeler can begin to be developed.

We have presented a parametric analysis of the impact of various trajectory forecasting error sources. We have applied error analysis to a sample collection of flights under various climbing, cruising and descending scenarios. Although this study does not comprehensively analyze all factors, a comparison of the data presented indicates that the pacing factor for trajectory forecasts depends on the DST application of interest and operational scenario to be considered. For this reason, attaining the goal of “common” trajectory modeling will, at a minimum, require the consideration of all errors presented herein.

Acknowledgements

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References


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**Figure 1.** Simple turn model (\(\phi\) = bank angle).

\[
R = \frac{V^2}{g \tan \phi} \quad R \tan \frac{\Delta \psi}{2} = \frac{1}{\cos(\Delta \psi / 2)} - 1
\]

**Figure 2.** Turn omission error.

**Figure 3.** Maximum Cross-track error distribution due to turn omission for flight plans and actual flights.

**Figure 4.** Along-track errors due to turn omission (mean plus one sigma) using actual turn distributions.

**Figure 5.** Altitude error due to weight error.
Figure 6. Along-track error due to weight error.

Figure 7. Scenario errors due to weight in climb.

Figure 8. Altitude errors due to gradient omission.

Figure 9. Along-track errors due to gradient omission.

Figure 10. Level off altitude histogram and cumulative duration distribution.

Figure 11. Standard errors due to level-offs.