REVIEW OF TRAJECTORY ACCURACY METHODOLGY AND COMPARSION OF ERROR MEASUREMENT METRICS

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Abstract

The aircraft trajectory is a prediction of the aircraft's anticipated flight path. Trajectory accuracy is measured by comparing the trajectory prediction to the actual flight path of the aircraft. Trajectory accuracy analysis starts with a sampling process that selects the actual and predicted trajectory positions for measurement and concludes with the application of statistical and graphical analysis methods. At the base of a study are the error measurements and how they are defined. Their definitions are the focus of this paper. Two spatial metrics and two time metrics were specifically defined and compared in the horizontal dimension. Both detailed synthetic flight examples and a large traffic sample were employed to evaluate these error metrics. The second spatial metric and first time error metric proved to be superior methods in turns and approximately equivalent elsewhere. From the traffic sample, the standard deviation of the differences between the presented spatial and time error metrics was significant, ranging from 0.6 to 4.2 nautical miles and 9 to 55 seconds, respectively. Therefore, it is concluded that it is essential to clearly define the particular error measurement technique applied in a trajectory accuracy study to not only be relevant, but also for the results to be extensible and cross comparable with other studies.

Introduction

The Federal Aviation Administration (FAA) has a variety of ground based air traffic management decision support tools (DSTs) to support the en route air traffic controllers. A fundamental component of a DST's design is the trajectory predictor, upon which its functionality is based. The trajectory predictor provides a prediction of the aircraft's anticipated flight path, determined from the flight plan and radar track data received from the National Airspace System (NAS) Host Computer System (HCS). The trajectory accuracy, or the deviation between the predicted trajectory and the actual path of the aircraft, directly impacts the overall accuracy of these automation tools.

It is difficult to predict these aircraft trajectories accurately. Several factors create this challenge, such as:

- Unknown winds aloft
- Redirection to avoid other aircraft or weather
- Holding patterns
- Flight plan revisions
- Unknown aircraft weight

For example, the pilot may request a different cruising altitude in an attempt to find a smoother flight or the controller may vector the aircraft to the right or to the left of his planned route to pass either behind of or on front of a conflicting aircraft.

Trajectory accuracy is measured by comparing a flight's predicted trajectory with the radar surveillance track reports of the actual route flown. The set of track reports that describe an aircraft's flight are referred to as the track. In the past, two FAA DST's that build trajectories and have undergone significant trajectory accuracy measurement are

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URET (User Request Evaluation Tool) and CTAS (Center TRACON Automation System). The URET developer, MITRE Corporation, has developed various performance metrics including trajectory accuracy metrics to evaluate URET's performance, reported in [1] and [2]. The CTAS developer, the National Aeronautical Space Administration (NASA), measured the trajectory accuracy performance of CTAS's Descent Advisor in the Denver Air Route Traffic Control Center (ARTCC) airspace, reported in [3] and [4]. The FAA's Conflict Probe Analysis Team (CPAT) at the FAA's William J. Hughes Technical Center (WJHTC) in its test and evaluation role measured the trajectory prediction accuracy of both these systems, reported in [5] and [6]. In these studies, the team developed and implemented spatial metrics and a novel sampling technique. Since these references were published, CPAT has continued the development of trajectory accuracy methods. In [7], CPAT applied inferential statistical methods to determine if a trajectory model had improved or degraded following a software upgrade.

This paper first highlights the overall process used to determine trajectory accuracy. Next, it focuses on a central component of trajectory accuracy, the error measurement itself. The paper compares two methods of measuring the spatial errors of aircraft trajectory predictions and two methods of measuring their time errors. The first spatial error metric, referred to here as "Time Matched Segment", has been applied in [5]. The first time error metric, referred to as "Closest Point", has been applied in [4]. The second spatial metric, referred to as "Closest Segment", is essentially the same as the one described in [1]. These metrics will be defined and compared using synthetic flight examples and results from an actual aircraft traffic sample.

Trajectory Prediction Accuracy Measurement

The basic components of trajectory accuracy measurement include:

- Sampling Process
- Measurement of Errors
- Statistical Analysis
  - Descriptive
  - Inferential
  - Context
- Graphical Tools and Other Considerations

Sampling Process

Researchers use different data sets for different testing objectives. Large data sets of observed trajectories are often collected to estimate the total error representative of the operational conditions (e.g., data typically collected from ATC automation using live traffic data). For this type of large-scale data collection, the analyst needs to choose the specific trajectory predictions (trajectories) and actual track data for evaluation. A sampling process is a systematic and unbiased procedure of selecting the trajectory and track positions to capture for error measurement. As described in detail in [5] and [6], CPAT's interval based sampling technique is a two-step process that defines how to pair the track and trajectory points. This technique may not be applicable for all studies.

Different smaller data sets are used to test specific sub-components of a trajectory predictor to allow the researcher to identify the error sources and possibly reduce them. For these smaller data sets, it may be more feasible to measure every trajectory and track position available. Experimental flight tests as flown in [4] are an example of this type of analysis.

For analyses where a plethora of recorded trajectory and track data is available as in [2] and [5], the interval based sampling technique is a very effective method for selecting the particular positions to sample, eliminating the need to measure every trajectory and track position. In [8], a study compared several sampling regimes and found the interval based sampling technique to be the most effective.

Since the interval based sampling technique will be applied later in the Experimentation Section of this paper, a brief description of the method will be presented. The large data set used in this study required a sampling technique because it was not feasible computationally to use all of the data. The first step of this technique is the selection of an aircraft and the sampling of track points in succession a parameter number of minutes apart (e.g. two minutes) until the end of the track is reached. Each track point selected as a sample has a specific time associated with it, referred to as the sample time. The aircraft's trajectories are then searched to find the most recent trajectory for the given sample time. This operation is repeated for every sampled track point for which available track and predicted trajectories exist. For some sampled track points, trajectories may not exist.

The second step samples future points on the selected trajectory relative to the current sample time. The first sampling step selects a point on the trajectory that has the same time value as the current track point, corresponding to a look-ahead time of zero seconds (i.e., the initial condition associated with this particular trajectory prediction). The second step selects points on the trajectory that are defined a parameter set of times into the future (e.g. 5, 10, 15, and 30 minutes). It then finds the future track reports that have the same times as the selected trajectory points. For each look-ahead time, errors are calculated between the selected trajectory points and their corresponding track points.

Measurement of Errors

After the sampling process systematically selects track positions and active trajectories, measurements are taken of the deviations between these positions (predicted compared to actual). Air traffic control (ATC) separates aircraft in space or in time or both. To capture space and time errors, a trajectory and a track can be compared either spatially or in terms of time.

The spatial errors are defined by dimension. The horizontal error is the straight-line difference in the horizontal plane between the actual track reported position of the aircraft and the time coincident trajectory predicted position. There are two approximately orthogonal components to the horizontal error: the longitudinal or along track error and lateral or cross track errors. The longitudinal error represents the along track distance difference between a track and its trajectory. The lateral error represents the side-to-side, or cross track, difference between a track point and its corresponding trajectory point. The vertical error is the difference between the time coincident reported track altitude and predicted trajectory altitude position.

Time error is the deviation in time between “along track” coincident track and predicted trajectory positions. The main focus of this paper is on various methods of calculating the time errors and the spatial along and cross track errors. These methods will be discussed in detail in later sections of this paper.

Statistical Analysis

The sampling and measurement process populates a database of the errors. This database is then queried for statistical analysis results. Descriptive statistics are used to estimate key population parameters. For example, the standard deviation of the predicted vertical error as a function of look ahead time provides an estimate of the variation of a trajectory modeler. This statistic describes the modeler’s ability to accurately predict the altitude position of an aircraft.

Inferential statistical tests are used to make a claim on the value of a population parameter or characteristic. For example, the horizontal error is the horizontal difference between the actual track reported position of the aircraft and the time coincident trajectory predicted position. As described in detail [7], it is often necessary for the developer to determine if the mean horizontal error has decreased between subsequent software releases of the trajectory modeler system. Thus, an inferential statistical test is applied to test this conclusion.

Context statistics are metrics that set the scope of the trajectory accuracy results. For example, an inferential statistic can be applied to determine if two trajectory modelers produce the equivalent mean horizontal error. This study requires both predictors to produce predictions for the same flights and flight time. Since the trajectory sampling process is focused on taking error measurements only when both the actual and predicted aircraft data exists, it is possible for the test to incorrectly conclude that one trajectory predictor is less accurate than the other (i.e., where the predictions of one predictor are available but not for the other). This can happen if the predictor with the lower mean horizontal error predicts aircraft trajectories that it can easily model, omitting the harder flights. Thus, an inferential statistical comparison would be biased against the other predictor that posts predictions for a greater range of operational conditions.

Context statistics alleviate the risk of such an error by measuring the number of valid flights modeled or the ratio of flight time the modeler had valid trajectories available. Therefore, context statistics serve to verify the scope of the other statistical tests and analyses.

Graphical Tools and Other Considerations

There are important implementation considerations in trajectory accuracy. It is convenient to use a relational database of error measurements to explore the causes of trajectory prediction inaccuracy. To allow this, the relational database should contain information on the various factors that influence a trajectory modeler's performance. Based on CPAT's experience, it is highly recommended to calculate all the sampled error measurements and tag them accordingly rather than excluding them based on given rules. For example, it is common practice for analysts to exclude trajectory errors that occur when aircraft are not adhering to their known air traffic controller clearances. As discussed in the earlier introduction, controller lateral vectors may cause significant deviations from the planned route. If the associated trajectory accuracy measurements are excluded entirely, a modeler's algorithms that detect these situations and appropriately model them cannot be evaluated. CPAT's recommended approach is to flag these situations and calculate the statistical analyses both with and without them.

Finally, a trajectory accuracy analysis would not be complete without graphical tools to examine the trajectory accuracy errors from all dimensions and perspectives. An example approach is presented in [9].

This section provided a brief overview of the key steps or components of an aggregate process for objective analysis of trajectory prediction accuracy. The subsequent sections will focus on one of these components: the error measurement. The next section will expand the error measurement definitions and present several techniques for calculating them.

Definitions of Trajectory Error Metrics

Spatial trajectory errors are the spatial differences of time coincident track and trajectory positions, while time errors are the time differences of spatially coincident track and trajectory positions. The horizontal and vertical spatial errors are well defined and commonly accepted. However, the components of the horizontal errors are not as well defined and there are alternate techniques of calculating them. In the subsequent sub-sections, two alternative techniques will be defined and compared.

Time errors are calculated by finding spatially coincident "abeam" track and trajectory positions, which is an approximate technique since points are often not exactly spatially coincident. In the subsequent sub-sections, two alternative techniques for calculating the time errors will be defined and compared as well.

Note, the coordinate system used in this study is a three dimensional rectangular (Cartesian) coordinate system. The track data and trajectory predictions are based on the stereographic coordinate system used by the Host Computer system.

Spatial Error Metric Alternative 1 – Time Matched Segment

This metric is illustrated in Figure 1, in which the aircraft is flying from left to right. As defined by the sampling process, the given track location or measurement point is designated as P1 in Figure 1. The trajectory is a set of straight-line segments defined by the waypoints – four are shown in the figure. The first step is to use interpolation to find the location on the trajectory that has the same time as the track point P1. This point is labeled P2. The horizontal error is defined as the straight-line distance between P1 and P2. The horizontal error will be the same for both Alternative 1 and 2, but the along and cross track errors will be different. The next step in calculating these measures is to select the next trajectory segment end point following P2, labeled as P3. A perpendicular is then drawn from the track point P1 to the trajectory segment P2-P3 or its line extension. In Figure 1, the perpendicular intersects the line P2-P3 extended at P4.

The cross track or lateral error is defined as the length of the line P1-P4. The along track or longitudinal error is defined as the length of the line P2-P4. As defined in [5], the track point to the right of the trajectory is a positive cross track error (otherwise negative); the track point ahead of the time synchronous trajectory point is a positive along track error (otherwise negative). The sign (positive or negative) of the cross track error is the same as the sign of the vector cross product of the vectors from P2 to P1 and P2 to P3. The sign of the along track error is the same as the sign of the scalar product of the same vectors.

Alternative 1 was first used in the investigation reported in [5], presented later in [6], and has continued to be used by CPAT for several FAA internal studies, such as documented in [10].

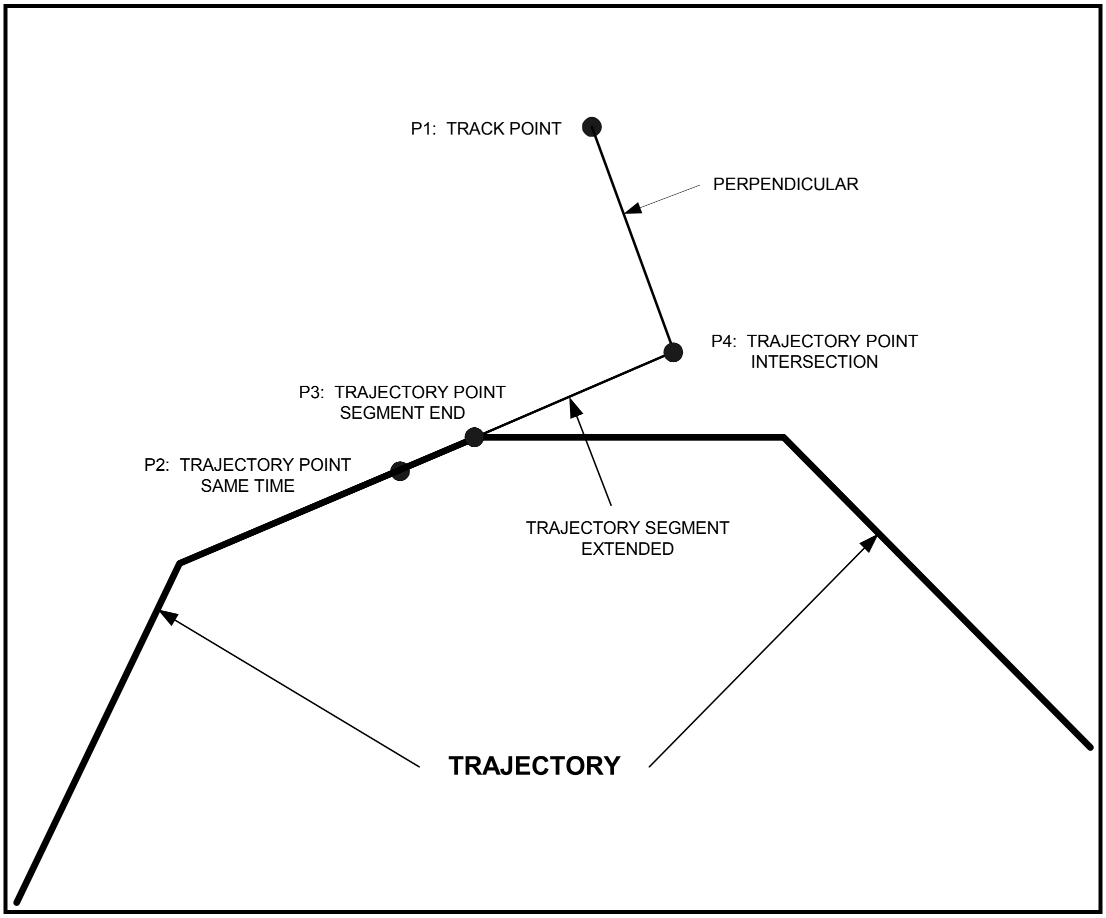

Figure 1: Track – Trajectory Geometry for Measuring Spatial Error Metric 1 – Time Matched Segment

Spatial Error Metric Alternative 2 – Closest Segment

Figure 2 illustrates the alternative, second spatial metric for the same track point and trajectory segments as Figure 1. The numbering of the key points is different however. The first step is to find the trajectory segment closest to the track point. In Figure 2, Q1 is the given track point and Q2-Q3 is the closest trajectory segment. In general, the closest segment is the one with the shortest perpendicular from the track point to the segment. However, if the perpendicular intersects the extension of the segment, the distance to the segment is not the length of the perpendicular, but is the distance from the track point to the nearer end of the segment. The segment with this minimum adjusted distance is the closest segment.

The cross track error is defined to be this minimum distance. In Figure 2, the perpendicular is shown to intersect the segment (not its extension) at point Q4. The cross track error is then the length of the line Q1-Q4.

To find the along track error, the time synchronous trajectory point is found by interpolation as is done in Spatial Error Metric 1 – Time Matched Segment. In Figure 2 this point is labeled Q5. The track point Q1 and the trajectory point Q5 have the same time coordinate. The along track error is the distance that Q4 is separated from Q5, measured along the trajectory. The distance along the trajectory is the sum of the lengths of the straight-line segments, which constitute the trajectory.

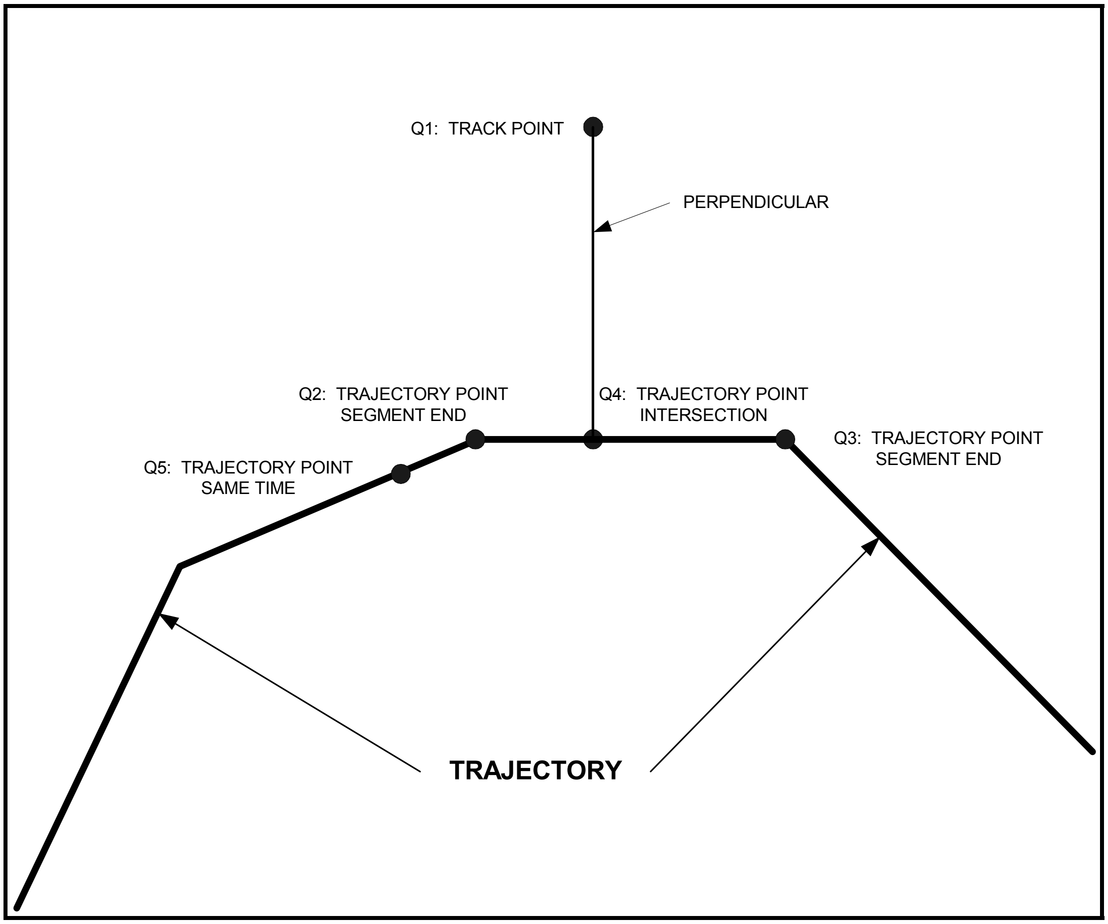

Figure 2: Trajectory Geometry for Measuring Spatial Error Metric 2 – Closest Segment - and Time Error Metric 1 – Closest Point

For purposes of this study, the sign of the errors follows the same convention as the previous metric. If Q4 is farther along the trajectory (later in time) than Q5, the along track error is positive (otherwise negative). The sign of the cross track error is found from the sign of the vector cross product of Q2 to Q1 and Q2 to Q3.

Dropping a perpendicular from the trajectory to the track instead of from the track to the trajectory is a third alternative measurement method that could be examined. However track measurement noise makes this method unsatisfactory.

<u>Time Error Metric Alternative 1- Closest Point</u>

As documented by CPAT in [11], and motivated by [12], this time error is the difference in times between the track point time and the time of the closest trajectory point. This metric is obtained similar to Alternative 2 of Spatial Error (Closest Segment). Referring to Figure 2, the time error is the difference in the time coordinates of Q1 and Q4 or ($t_4 - t_1$). The time error is defined here as positive when the track is ahead of the trajectory. For example, if the aircraft gets to the sampled point sooner than the trajectory prediction, the time error is positive.

The calculation of this time error metric requires a trajectory point that matches the sampled track point in space. The two methods of calculating the spatial error described above required finding a trajectory point that has matched the sampled track point in time. Matching points in time is straightforward because there is only one coordinate to match (the time field) and the only way to match them is to make them equal. Matching points in space is more complicated because there are two coordinates and a choice of definitions when matching them.

For Time Error 1 (Closest Point), the matching trajectory point is defined to be the point closest to the sampled track point in the horizontal plane. A different technique of a spatially matching trajectory point is used in the measurement method described next.

Time Error Metric Alternative 2 – Trajectory Following
As before, a trajectory point is found which matches in space the sampled track point, this time based on coincident lengths of distance flown. The time error is the difference of the time coordinates of the track point and this matching trajectory point. The matching method is illustrated in Figure 3.

In this measurement method, called "Trajectory Following", the matching trajectory point is the one that has the same along track distance as the sampled track point. Since the along track distance is relative to some reference point, a reference point is needed for both the track and the trajectory. The spatial matching problem is then to find a pair of matching points – one point on the track and one point on the trajectory – to initialize the distance flown computation. These points are called the anchor points for the track and the trajectory. The first point of the trajectory is selected as the candidate anchor point for the trajectory. This first trajectory point corresponds to the track point at the time of trajectory generation (i.e., t_0 for a specific trajectory prediction).

With the selection of a candidate trajectory anchor point, a search is made for the track point closest to this first trajectory point. Usually a track point can be found within a nautical mile of the first trajectory point. The trajectory point matching the sampled track downstream point is the trajectory point which is the same distance flown along the trajectory from the trajectory anchor point as the track point is from the track anchor point.

Referring to Figure 3, the sampled track point is R1, the first trajectory point is R2 and is the trajectory anchor point. The track point closest to R2 is R3. R3 is the track anchor point. The along track distance flown from R3 to R1 is calculated. The trajectory point R4 is defined as the point with the same along track trajectory distance from the trajectory anchor point R2. Thus, the time error is the difference of the time coordinates of the points R4 and R1 or (t_4-t_1).

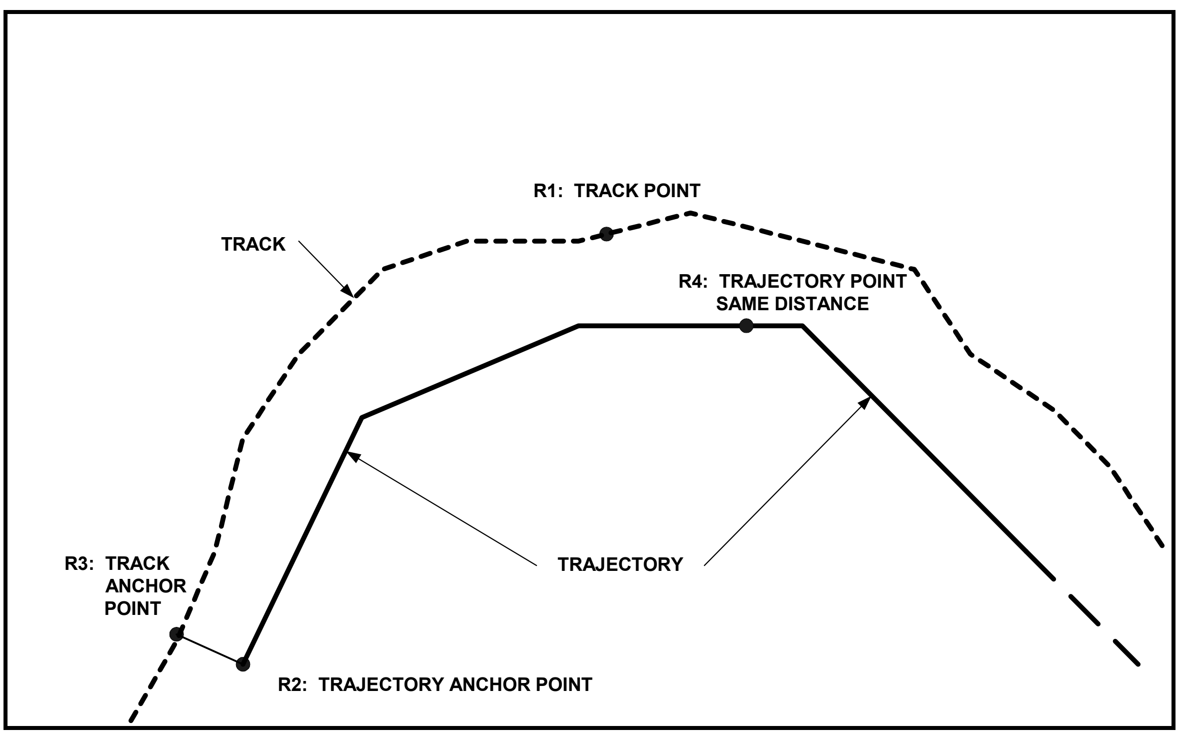

Figure 3: Track-Trajectory Geometry for Measuring Time Error Metric 2 – Trajectory Following

If there is no track point within the threshold distance of the first trajectory point, the search is extended to the second trajectory point, then to the third point, until a pair of trajectory and track points are found whose separation is less than the threshold value. If no pair is found, the closest pair of track and trajectory points is used as the pair of anchor points. (Alternatively, the track point could be interpolated to find a track anchor point abeam the trajectory anchor point.) This occurs very rarely.

Experimentation

A study was conducted to compare these metrics using both synthetic and recorded field data. The synthetic data was chosen to illustrate the differences among the measurements with the track to trajectory differences accentuated. About 30 minutes of synthetic data was generated in a straight line path and then again in a right turn. For the recorded field data experimentation, seven hours of air traffic data from the Fort Worth Air Route Traffic Control Center (ZFW ARTCC) was applied. This data was originally used in [5]. It is composed of over 2300 flights with 440,000 track position reports and over 352,000 aircraft trajectories produced by the CTAS DST.

Synthetic Test Data for Straight Flight

A portion of a straight and level aircraft flight path and its trajectory prediction was synthesized to compare the values of the error metrics. This data is shown in Figure 4. The lower line is the predicted trajectory, which is defined by three points. The upper line is the track. The first point of the track segment, Track Point 1, and the first point of the trajectory segment, Trajectory Point 1, are time coincident at a time of 25,200 seconds UTC. Similarly, the last track segment point, Track Point 9, and the last trajectory segment point, Trajectory Point 3, are time coincident at a time of 27,120 seconds UTC.

The aircraft flies a route offset to the left of the predicted trajectory by 40 nautical miles and is longitudinally ahead of the trajectory by 40 nautical miles, which at 450 knots is equal to 320 seconds. This synthetic aircraft flies at a constant speed (450 knots), which is correctly modeled in the predicted trajectory.

Error measurements were made for each of the nine track sample points. A total of six error measurements were made for each of the sample points. Two measurements for each of the spatial metrics (i.e. along and cross track errors) and one each for the time error metrics were applied. For comparison, the spatial and time errors are listed in Table 1 and 2, respectively.

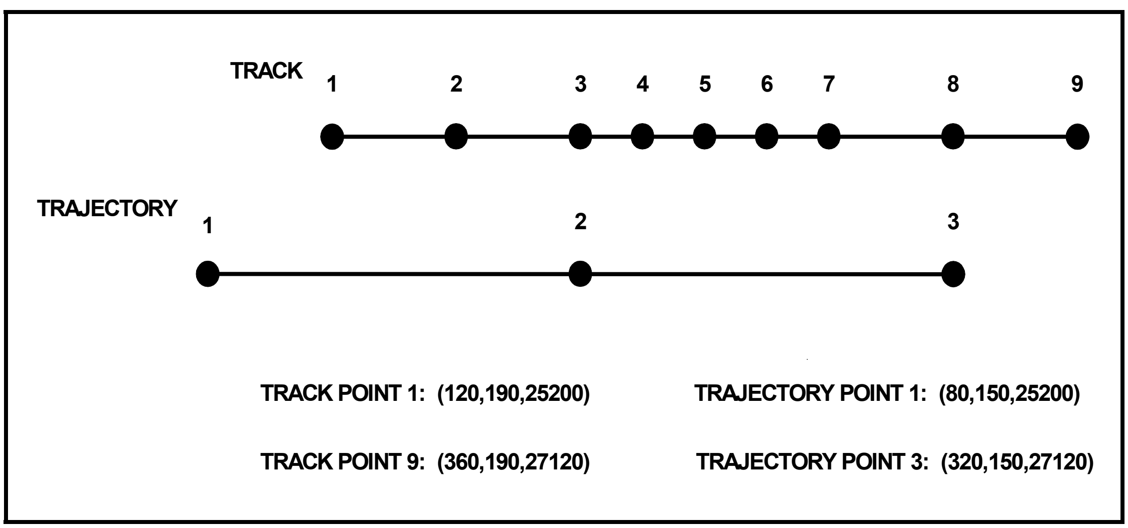

Figure 4: Straight Flight of Track and Trajectory

Except for the last sample point, both the spatial and time error measurement metrics obtain the same error values, respectively. The values differ for sample track point 9 and the Time Error 2 (Trajectory Following) is undefined due to the edge effects caused by the finite length of the trajectory. In this example, Spatial Metric 2 (Closest

Segment) attempts to project a line onto the trajectory from track point 9, but the trajectory ends and the projection falls beyond the end of trajectory point 3. As previously discussed in the Section *Definitions of Trajectory Error Metrics*, if the projection is off the line segment, the distance to the closest end point of the segment is used. In this example, the cross track error at trajectory point 3 is 57 nautical miles and is larger than the previous errors. The next synthetic test case will show that the various methods obtain appreciably different results when there is a turn in the track and trajectory.

Table 1: Straight Track - Cross Track and Along Track Trajectory Errors

SAMPLE TRACK POINT	CROSS TRACK ERROR 1 NM	CROSS TRACK ERROR 2 NM	ALONG TRACK ERROR 1 NM	ALONG TRACK ERROR 2 NM
1	-40	-40	40	40
2	-40	-40	40	40
3	-40	-40	40	40
4	-40	-40	40	40
5	-40	-40	40	40
6	-40	-40	40	40
7	-40	-40	40	40
8	-40	-40	40	40
9	-40	-56.6	40	0

Synthetic Test Data for a Turn in Flight

Illustrated in Figure 5, the data in this example is the same as in the first example except a 90-degree turn has been added to the track and the trajectory. The track follows a route on the outside of the turn predicted by the trajectory. Again the track is offset to the left of the predicted trajectory by 40 nautical miles and is 40 nautical miles ahead.

Figure 5's time values for both the track and the trajectory are the same as in the previous example. The ends of the track and trajectory segments are time coincident as before. The trajectory has an instantaneous turn at the waypoint. This is unlike the method used by NASA Ames in [4] and illustrated in [12], which models turns by circular arcs that are a more realistic representation of the actual flight path of an aircraft. However, this example is adequate to demonstrate the differences in the alternative metrics.

The trajectory errors for this example are given in Tables 3 and 4. The cross track errors as measured by the Spatial Metric 1 (Time Matched Segment) are 40 nautical miles (nm), which is the same as for the straight flight example. The cross track errors for the Spatial Metric 2 (Closest Segment) shows an increase as the aircraft swings wide around the turn – a more accurate measurement. The along track errors using Spatial Metric 1 (Time Matched Segment) are positive 40 nm before the turn and then abruptly switch to negative 40 nm after the turn. The along track errors using Spatial Metric 2 (Closest Segment) start out at positive 40 nm before the turn and gradually change to negative 40 nm after the turn – again a more accurate measurement.

Before the turn, the Time Metric 1 (Closest Point) shows that the track is ahead of the trajectory by 320 seconds, gradually falling behind the trajectory through the turn, and ending up behind the trajectory by 320 seconds. This is a realistic representation of the time error. The Time Metric 2 (Trajectory Following) gives a constant time error of 320 seconds, a measurement of little relevance to ATC applications. The reason for the second Time Metric's inaccuracy is it does not update its measurement to accommodate a turn unless a new trajectory with a new anchor point is selected.

Table 2: Straight Track – Trajectory Time Errors

SAMPLE TRACK POINT	TIME ERROR 1 SECONDS	TIME ERROR 2 SECONDS
1	320	320
2	320	320
3	320	320
4	320	320
5	320	320
6	320	320
7	320	320
8	320	320
9	0	-

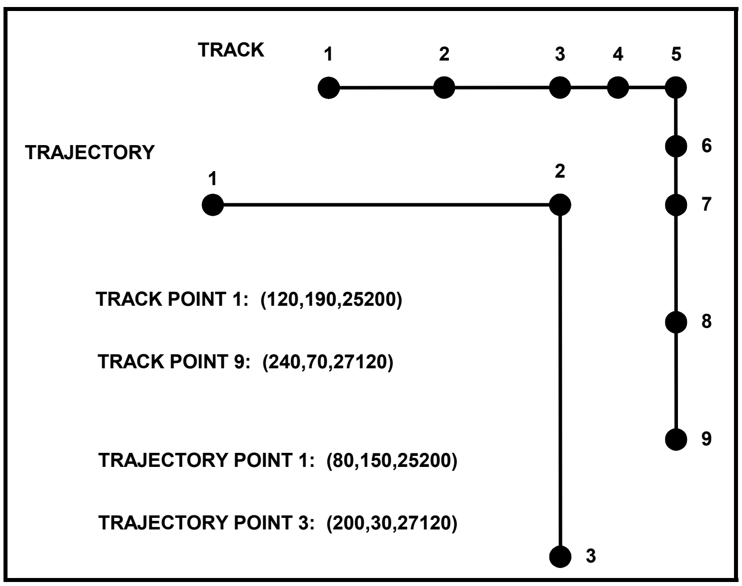

Figure 5: 90-Degree Turn Flight of Track and Trajectory

Actual Air Traffic Test Data

The six trajectory accuracy metrics were applied to seven hours of actual air traffic data from ZFW. This same traffic sample was collected in January 1999 for an earlier study documented in [5]. For the current application, the interval based sampling technique, as previously defined, was employed using a two-minute sampling interval and iterated at each sample with five-minute look-ahead times up to twenty minutes in the future. This resulted in approximately 140,000 measurements taken for each of the six techniques.

Table 3: 90 Degree Turn - Cross Track and Along Track Trajectory Errors

SAMPLE TRACK POINT	CROSS TRACK ERROR 1 NM	CROSS TRACK ERROR 2 NM	ALONG TRACK ERROR 1 NM	ALONG TRACK ERROR 2 NM
1	-40	-40	40	40
2	-40	-40	40	40
3	-40	-40	40	40
4	-40	-44.7	40	20
5	-40	-56.6	-40	0
6	-40	-44.7	-40	-20
7	-40	-40	-40	-40
8	-40	-40	-40	-40
9	-40	-56.6	-40	-40

Table 4: 90 Degree Turn - Trajectory Time Errors

SAMPLE TRACK POINT	TIME ERROR 1 SECONDS	TIME ERROR 2 SECONDS
1	320	320
2	320	320
3	320	320
4	160	320
5	0	320
6	-160	320
7	-320	320
8	-320	320
9	-320	-

Each of the three metrics (along track, cross track, and time error) was applied, using both spatial and both temporal approaches. These metrics were then compared by taking their differences. For the along track spatial metrics, the signed Along Track Metric 2 was subtracted from the signed Along Track Metric 1. Similarly the Cross Track Metric 2 was subtracted from the Cross Track Metric 1 and the same for the Time Metrics. Three sets of differences or residuals were calculated. The following Equations 1, 2, and 3 define these residuals, respectively.

$$\Delta\, Along_i = AlongMetric1_i - AlongMetric2_i \qquad (1)$$

$$\Delta\, Cross_i = CrossMetric1_i - CrossMetric2_i \qquad (2)$$

$$\Delta\, Time_i = TimeMetric1_i - TimeMetric2_i \qquad (3)$$

where i is the index for matching pairs of trajectory and track points

The sample means and standard deviations of these residuals were calculated for different look-ahead times. For the spatial along and cross track metrics, the sample means ranged from negative 0.12 to positive 0.06 nautical miles respectively, for the 0 to 20 minutes look-ahead times. For the time metrics, the sample means ranged from about 0 to 10 seconds. However, the variability as quantified by sample standard deviation was much larger. The Equation 1 and 2 spatial metric residuals were plotted in Figure 6 for look-ahead times 0 to 20 minutes. For the along track metrics, the standard deviations of the residuals ranged from approximately 0.9 to 4.2 nautical miles. For the cross track metrics, the standard deviations of the residuals were slightly lower, ranging from approximately 0.6 to 3.5 nautical miles. When typical errors are between 0 and 8 nautical miles, these differences in the error metrics themselves are substantial.

The standard deviations of residuals for the compared time metrics are plotted in Figure 7. The results vary from 6 to approximately 55 seconds. Once again, for the two compared time metrics the variation in resulting measurements is substantial.

From the synthetic flight examples, the metrics did not differ by much in straight flight but did have significant differences during turns as illustrated in the measurements in Tables 1-4. For the actual traffic data, further analysis was performed on the 352,000 predicted trajectories sampled to determine the distribution of turn angles between flight segments along the trajectory. Of the seven million trajectory segment end points produced, 97 percent had turn angles less than ten degrees. Thus, for the overwhelming majority of the sample data, the aircraft are flying relatively straight, making the sample means of the residuals close to zero. The individual trajectory differences are signed and consequently the positive differences are cancelled in part by the negative differences and vice versa. This effect also tends to make the means close to zero. The positive and negative values do not cancel in the sample variation and standard deviation calculation. Since the standard deviation statistic squares its input, the signed differences from Equations 1-3 become unsigned.

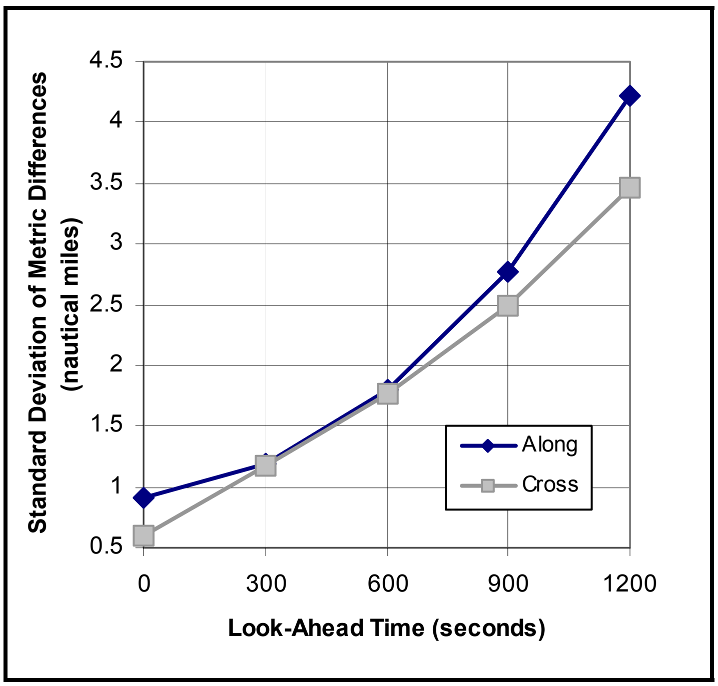

Figure 6: Standard Deviation of Compared Spatial Metric Residuals

In the data processed, the expected values of the difference metrics are over thirty-five times less than the variability (standard deviations) of the differences between the compared metrics. In other words, the measurements may not differ by much most of the time, but when they do, the differences are substantial.

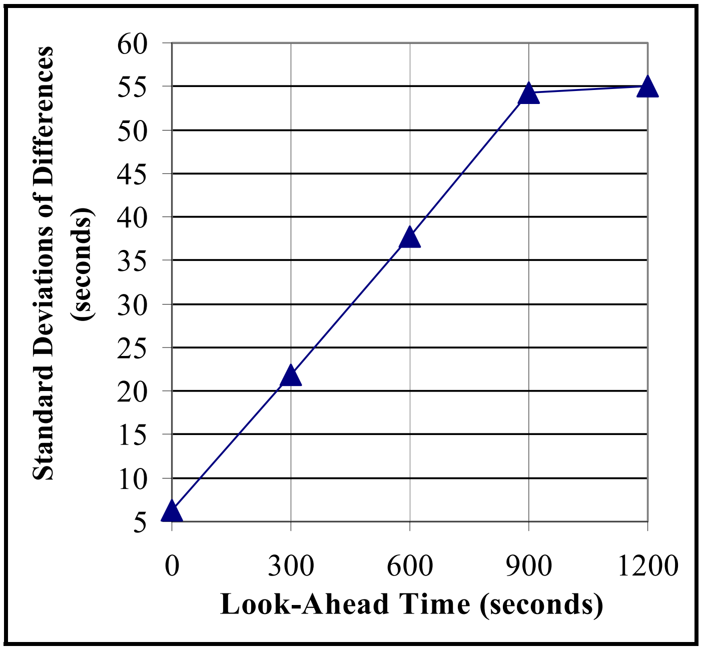

Figure 7: Standard Deviation of Compared Time Metric Residuals

Conclusions

An overview of the key steps or components of an objective trajectory accuracy measurement process was presented. The process starts with a sampling method that is applied to select the actual track and predicted trajectory positions from large sets of traffic data in an unbiased manner, such as the interval-based sampling method. Next, the error measurements are taken from these selected positions and stored in a relational database for later statistical and graphical analysis. Descriptive and inferential statistics are then calculated to estimate key parameters of the error measurements and make conclusions or claims about them. Context statistics are then used to verify the scope these results. Finally, graphical tools are used to review the data for given flights for several dimensions, allowing the analyst to explain the statistical results.

The underlying component of the trajectory accuracy process is the set of specific metrics used to capture the trajectory errors. Two spatial metric alternatives and two time metric alternatives were specifically defined and compared in the horizontal dimension. Both detailed synthetic flight examples and a large traffic sample were used to evaluate these error metrics. The Alternative 2 (Closest Segment) spatial along and cross track errors was shown to be a superior method during aircraft turns and both techniques were equivalent elsewhere. Similarly, Alternative 1 Time Error Metric (Closest Point) proved to the better technique over the second Time Error Metric (Trajectory Following).

While the average differences between metrics as defined in Equations 1-3 were relatively small and under 0.2 nautical miles, the variability of the results from the traffic sample were not. As illustrated in Figure 6, the standard deviation of differences between spatial error metrics ranged from 0.6 to 4.2 nautical miles. Figure 7 illustrates the differences between the two presented time error metrics, which ranged from 9 to 55 seconds. Therefore, the particular metric used to perform the error measurement in a trajectory accuracy study can produce significantly different results. For this reason, it is highly recommended when reporting trajectory accuracy results to clearly define the error measurements applied.

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