Session 35-GNC-21: "Innovations and Support of the NAS at the FAA"

Paper AIAA-2007-6517: "Implementation and Metrics for a Trajectory Prediction Validation Methodology"

Presented to: AIAA GNC, Hilton Head, S.C.

By: Mike Paglione, FAA

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Federal Aviation Administration

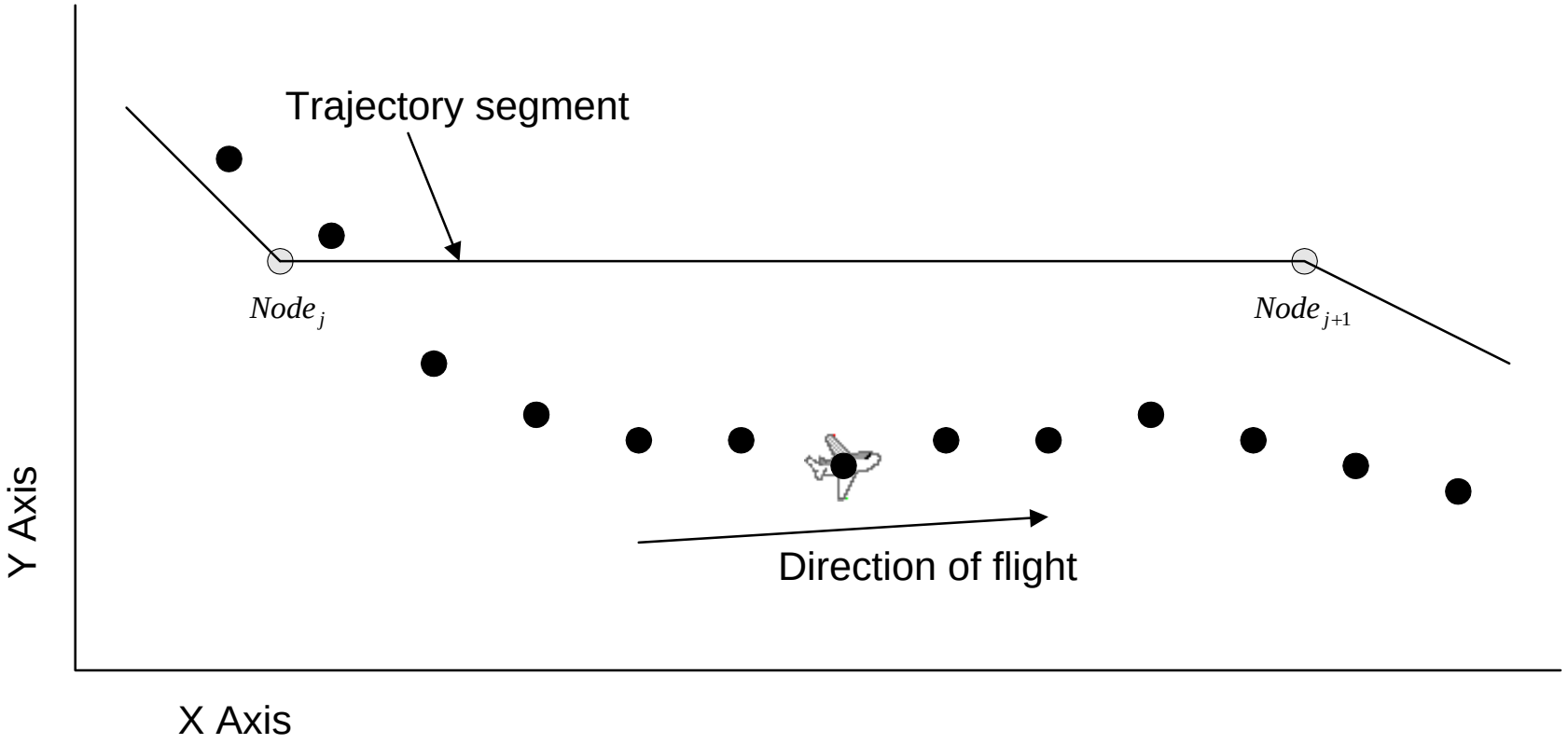
Overview

- **Motivation**
- **Methodology Topics**
  - Interval Based Sampling Technique (IBST)
  - Metrics
  - Analytical Methods
- **Implementation Considerations**
- **Application Example**

Motivation

- **FAA developing and deploying decision support tools (DST) to aid the air traffic controller**
- **DSTs have many functions, such as:**
  - Conflict probe
  - Metering and spacing
  - Air traffic advisory
- **Fundamental to all these DSTs is the trajectory predictor (TP)**
  - Aircraft trajectory – actual or prediction of the four-dimensional path of the aircraft
  - Each DST has a required level of accuracy for its trajectory predictions

Actual Versus Predicted Trajectory

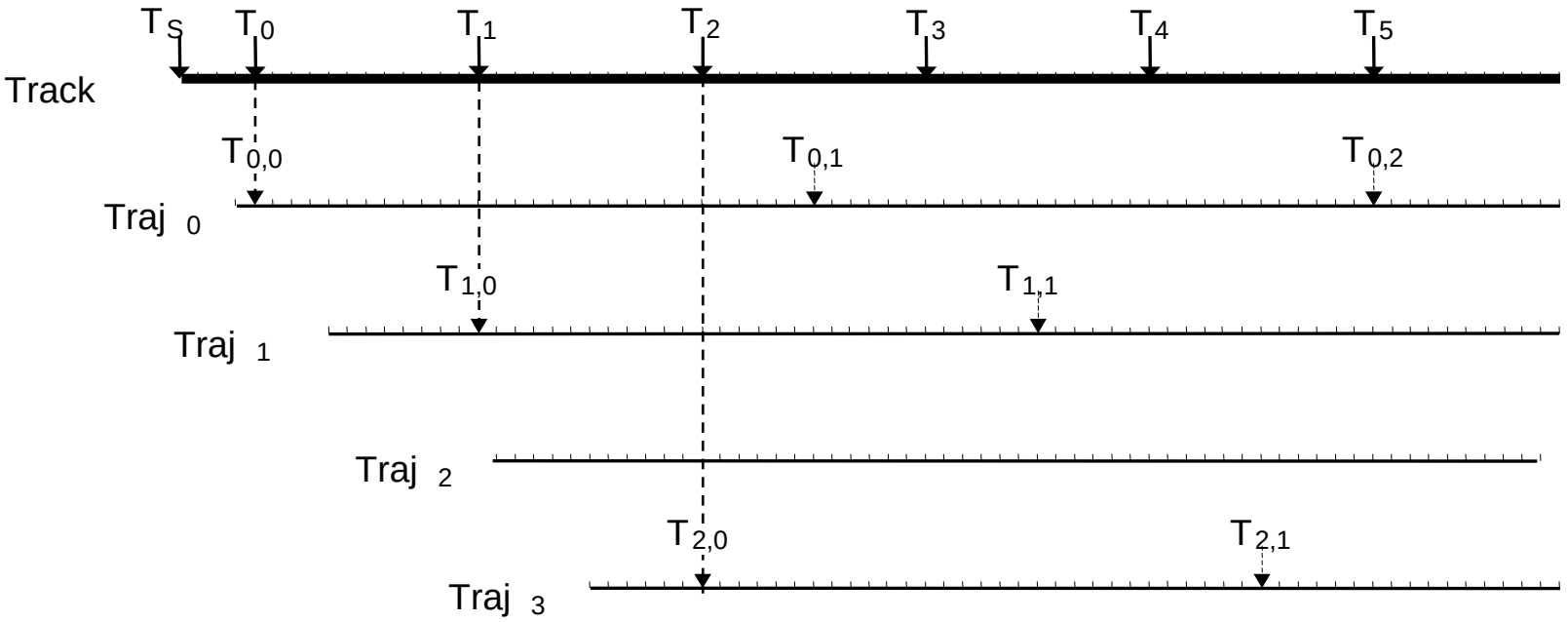


Methodology Topics

- ***Interval Based Sampling Technique***
- **Metrics**
- **Analytical Methods**

Interval Based Sampling Technique (IBST)

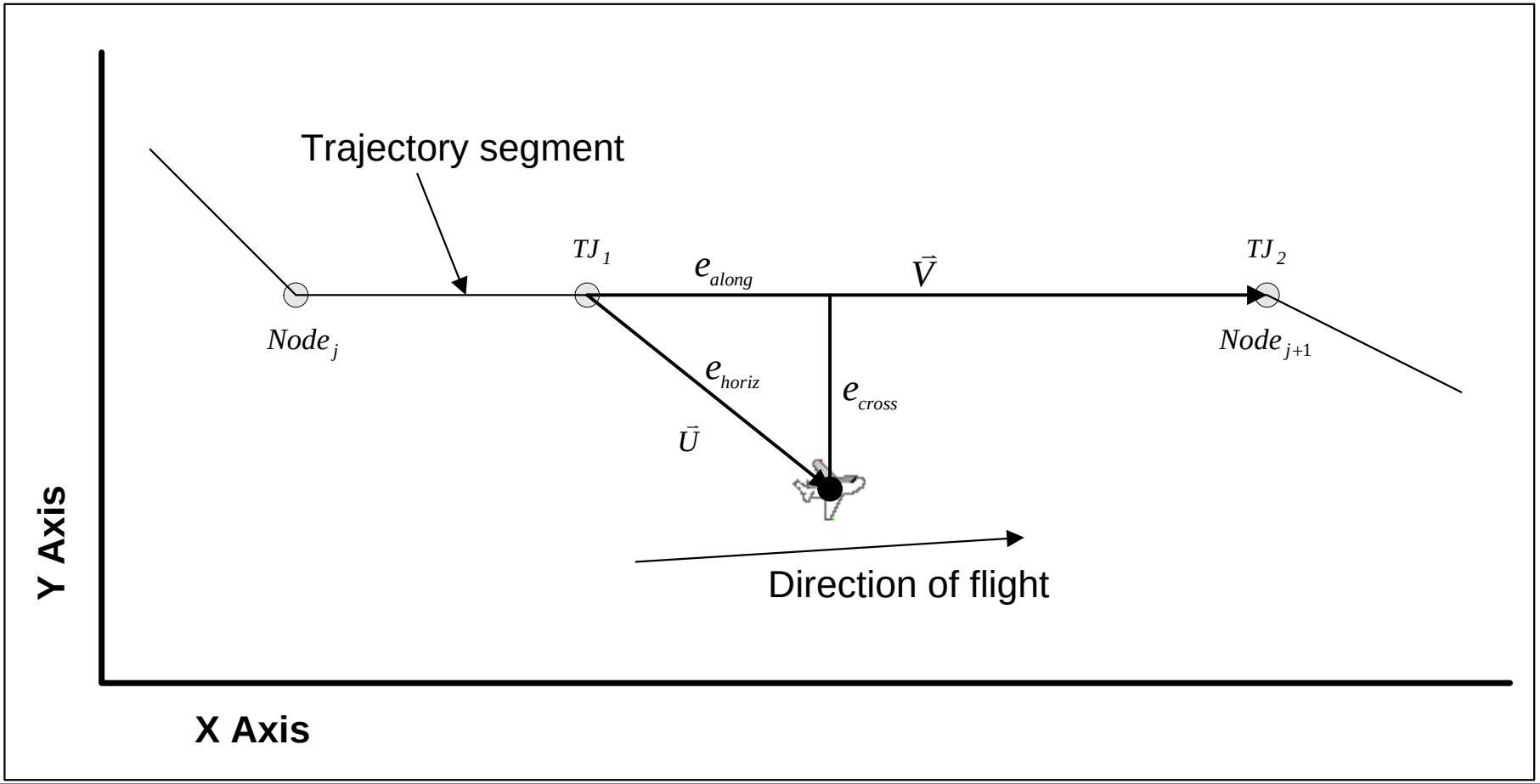
- **Using a constant sampling interval over track positions select active trajectory**
- **Then take measurements at various look-ahead (LH) times**
- **Aggregate results into statistics at each LH or display via GUI**



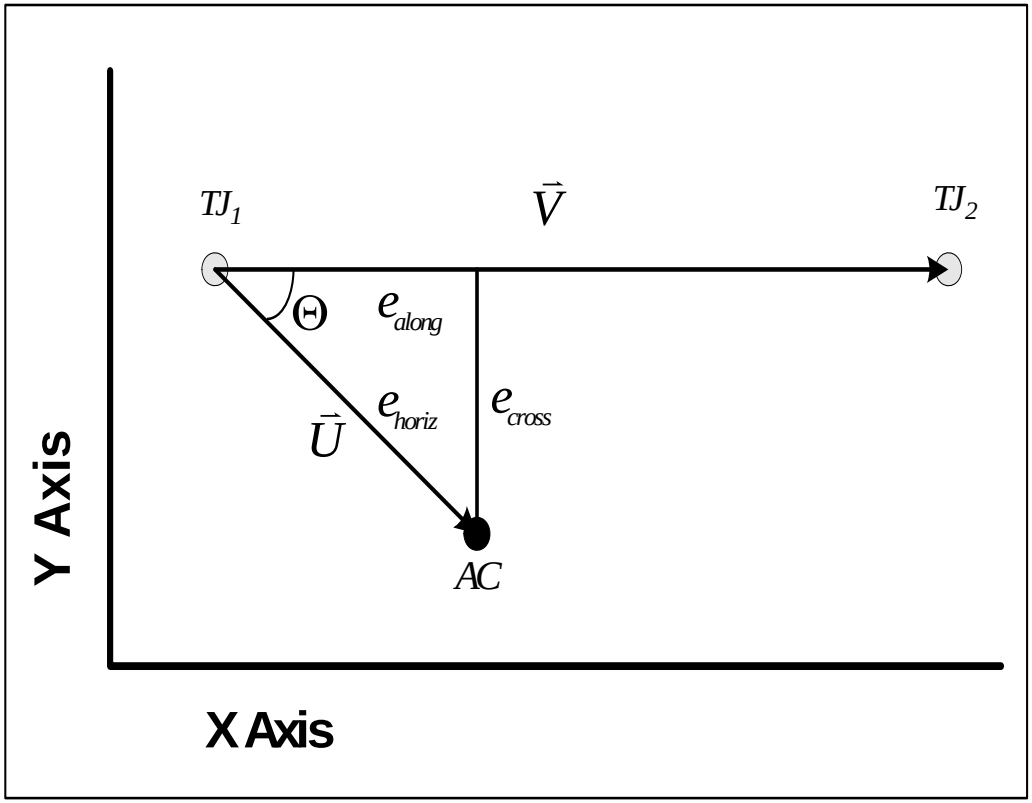
Methodology Topics

- **Interval Based Sampling Technique**
- ***Metrics***
- **Analytical Methods**

Trajectory Accuracy Metrics: Horizontal Dimension



Trajectory Accuracy Metrics: Horizontal Dimension

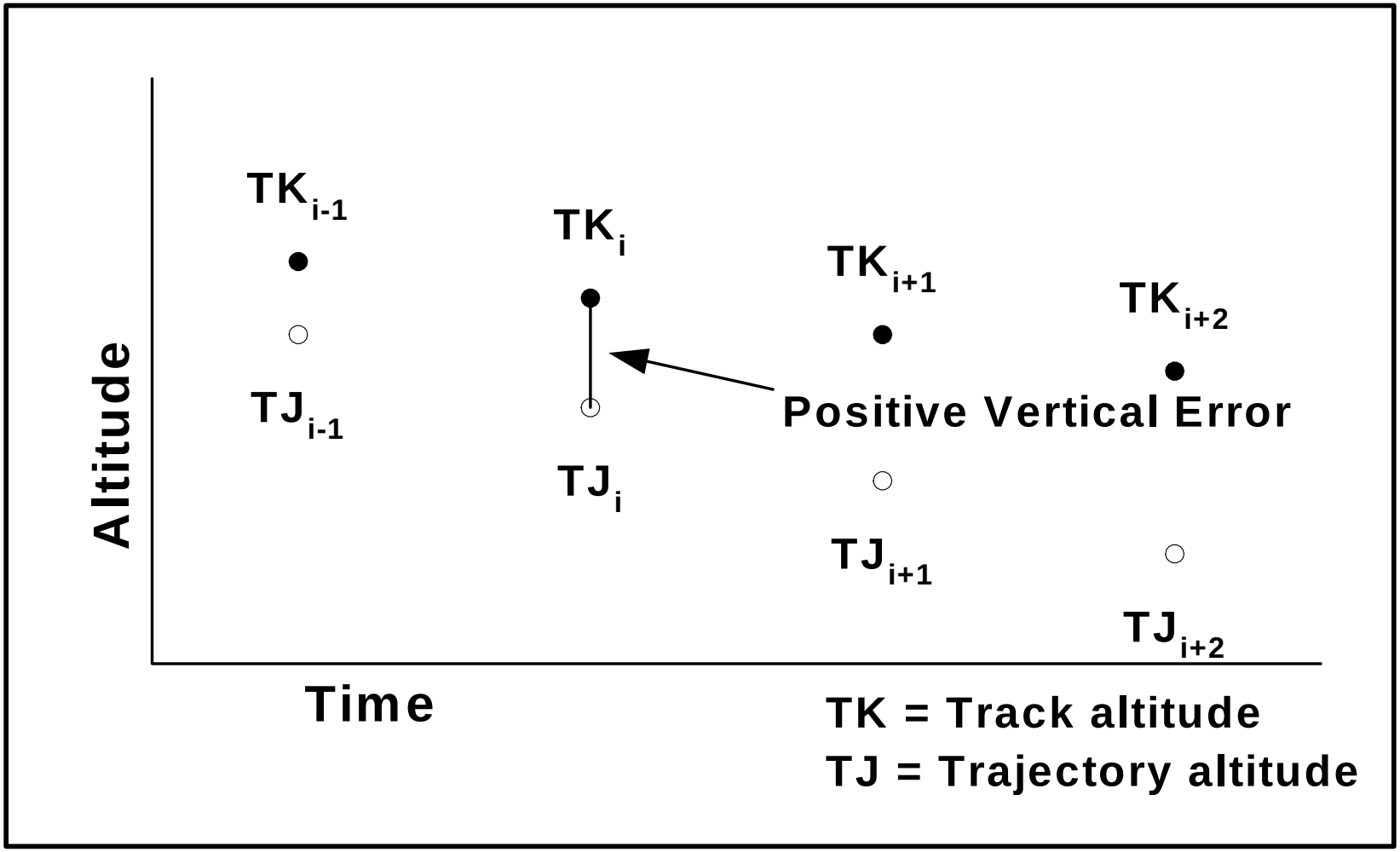


$$e_{horiz} = \left|\vec{U}\right| = \sqrt{u_x^2 + u_y^2}$$

$$e_{along} = \frac{\left(\vec{U} \bullet \vec{V}\right)}{\left|\vec{V}\right|} = \frac{u_x v_x + u_y v_y}{\sqrt{v_x^2 + v_y^2}}$$

$$e_{cross} = \left|\vec{U}\right| \sin\Theta = \frac{\left(\left|\vec{U} \times \vec{V}\right|\right)}{\left|\vec{V}\right|} = \frac{u_x v_y - u_y v_x}{\sqrt{v_x^2 + v_y^2}}$$

Trajectory Accuracy Metrics: Vertical Dimension



Methodology Topics

- **Interval Based Sampling Technique**
- **Metrics**
- ***Analytical Methods***

Statistical Hypothesis Test

- **Statistically manage the tradeoff between two basic test errors:**
  - Rejecting a system that actually is good (Type I)
  - Accepting a system that actually is bad (Type II)
- **How to properly balance these errors**
  - Can account for measurement variability and sample size
  - More is better – more measurements you have the higher confidence in your trajectory accuracy measurements

Applying the Hypothesis Test (e.g. as a Regression Test)

- **Assumes a null hypothesis**
  - Baseline system has a mean trajectory error, μ_b
  - New system has a mean trajectory error, μ_n
  - Null hypothesis:
    - The means are equal
    - No change in accuracy between the baseline and the new release

$$H_o : \mu_b - \mu_n = 0$$

- **Typically, alternative hypothesis is the new system mean is greater than the baseline mean**

Lesson Learned - Pair the Data!

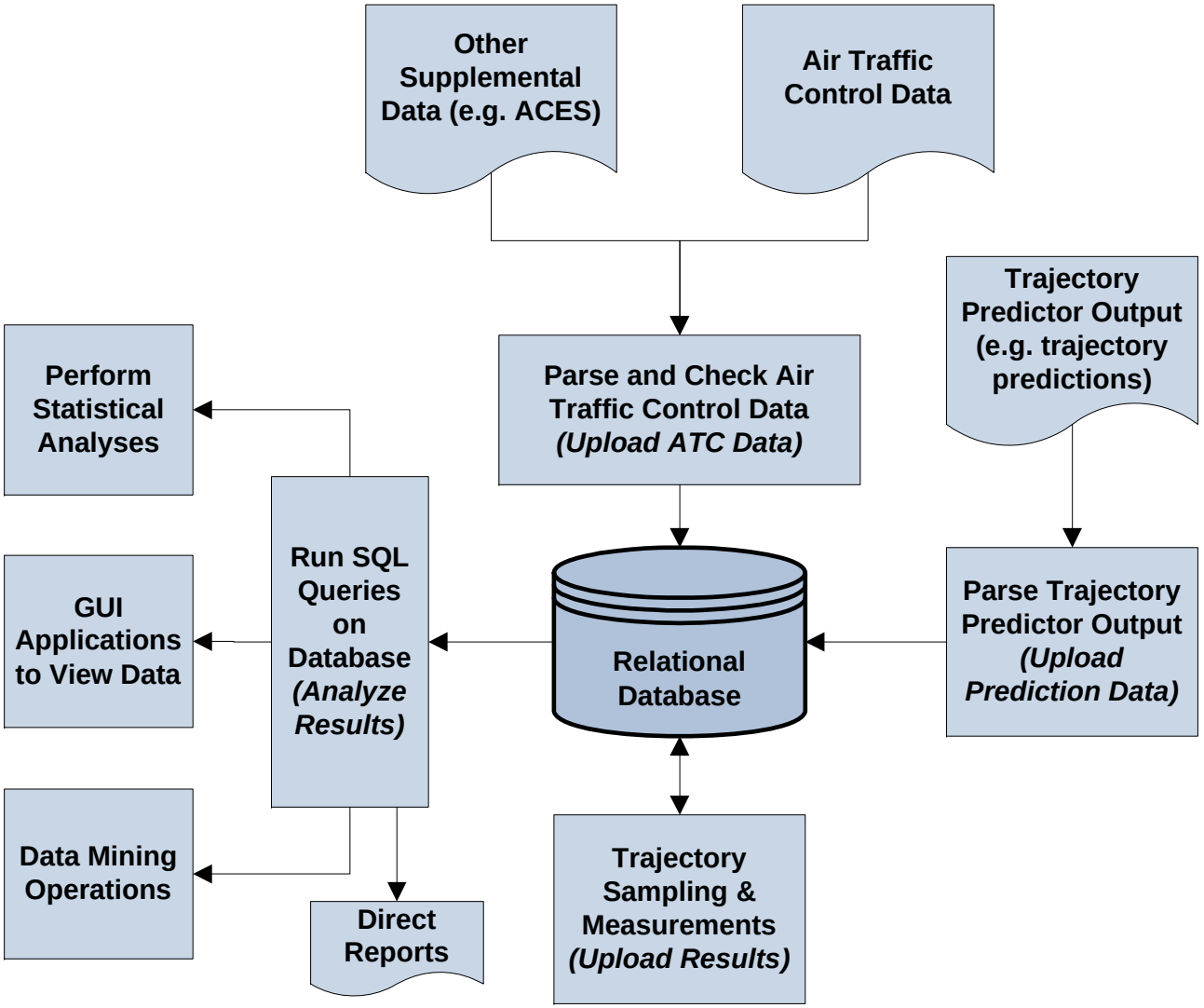
- **Usually test compares means by assuming independent runs**
- **Regression tests use the same input data, so test runs are not independent!**
- **However each trajectory measurement can be paired**
- **Pairing**
  - Handles the independence assumption
  - Reduces the sample variance allowing fewer samples
  - Does reduce sample size by half
- **Test statistic**

$$D_i = B_i - N_i$$

$$t = \frac{\bar{d}}{s_D / \sqrt{n}}$$

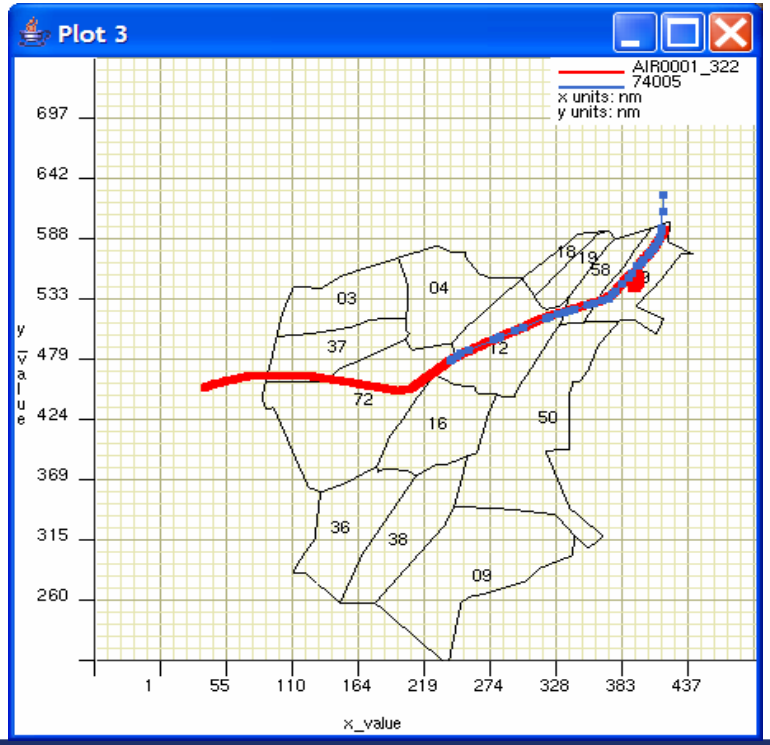
$$\text{Reject null hypothesis if } t \le -t_{\alpha, n-1}$$

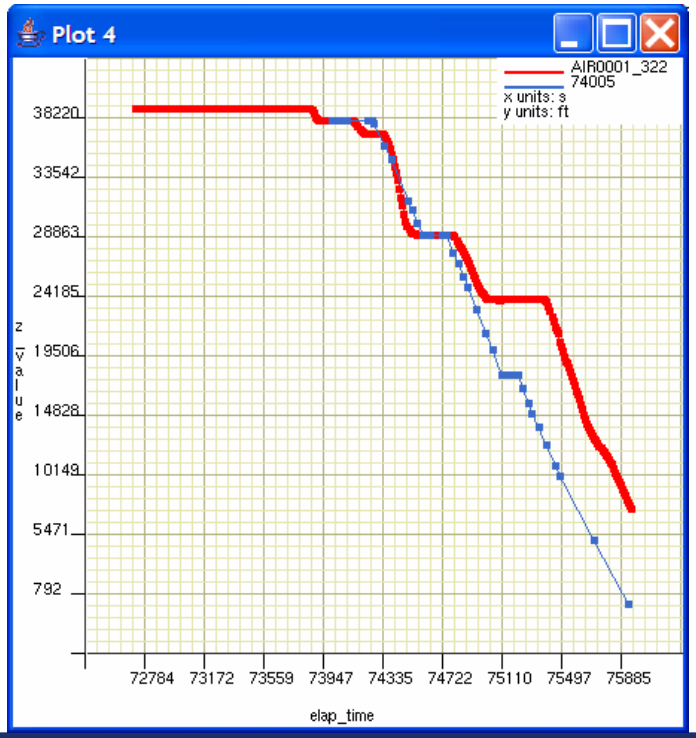
Implementation – Use of Database



Sample Flight

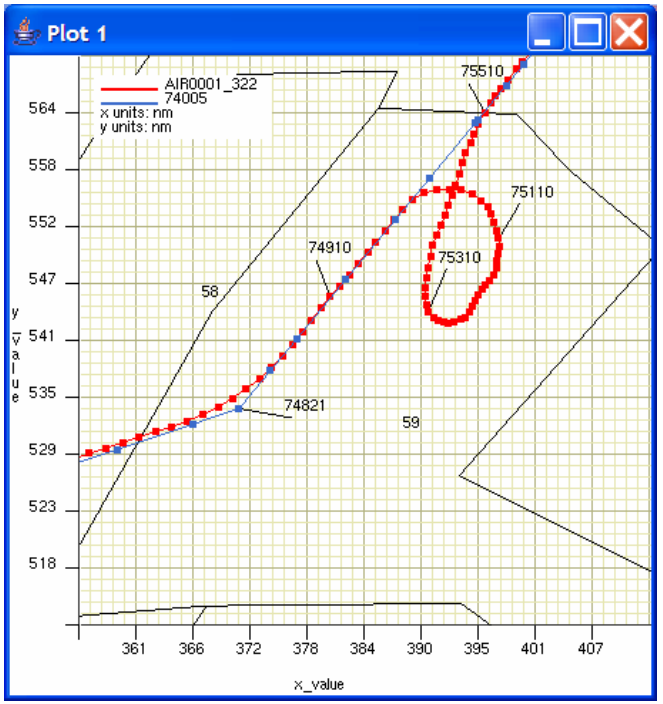
- Civilian airline over flight in Washington ARTCC (ZDC)
- Origin: Dallas Fort Worth, Texas
- Destination: John F. Kennedy International Airport, New York
- Hand-off into ZDC at 20:14 UTC and outbound to New York ARTCC at 20:56 UTC during brief cruise at FL 240





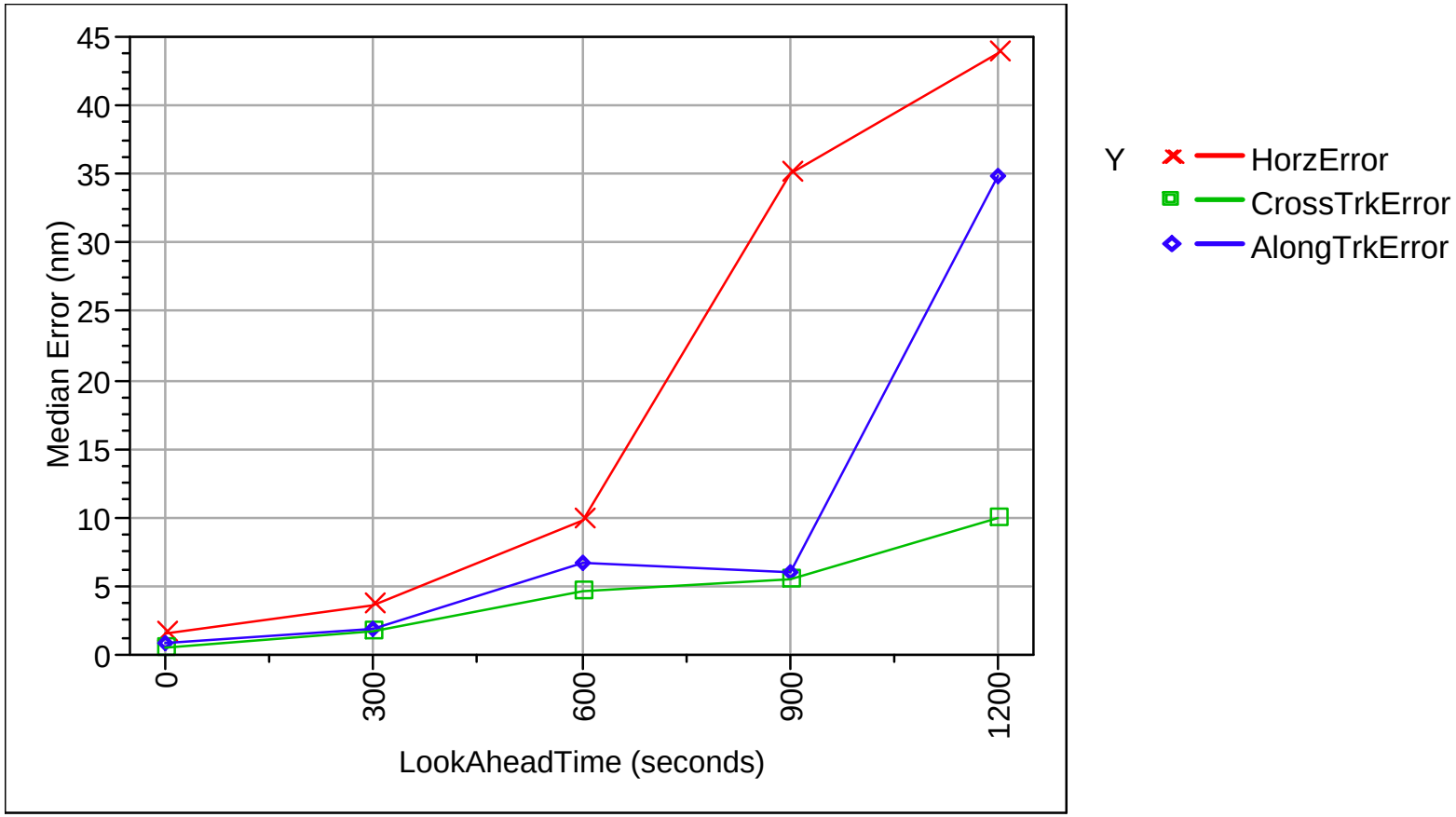
Sample Flight Results

- TP generated 34 trajectories
- IBST sampled 18 to produce 109 measurements
- Focus on trajectory build 74005 seconds (20:33:25 UTC) with 5 measurements below



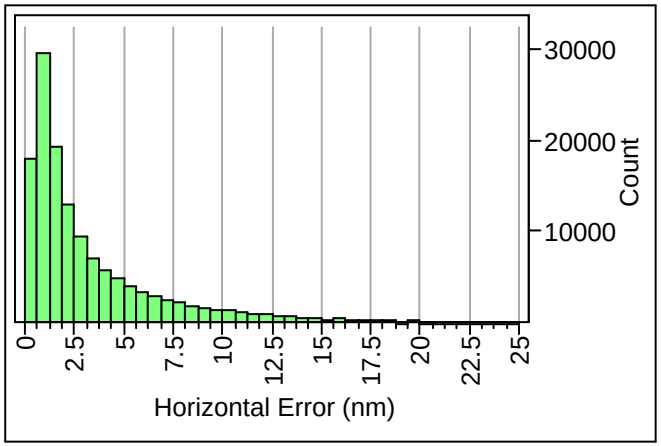
Sample Time	Measurement Time		Look Ahead Time	Horizontal Error	Cross-track Error	Along-track Error	Vertical Error	Clear Flag
Seconds	Seconds	HH:MM:SS	Seconds	Nautical Miles	Nautical Miles	Nautical Miles	Feet	
74040	74040	20:34:00	0	0.4	0.3	-0.3	0	0
74040	74340	20:39:00	300	0.1	-0.1	0.0	793	1
74040	74640	20:44:00	600	1.2	-0.5	-1.0	0	1
74040	74940	20:49:00	900	2.1	-0.1	2.1	2096	1
74040	75240	20:54:00	1200	34.6	11.9	-32.5	6952	1

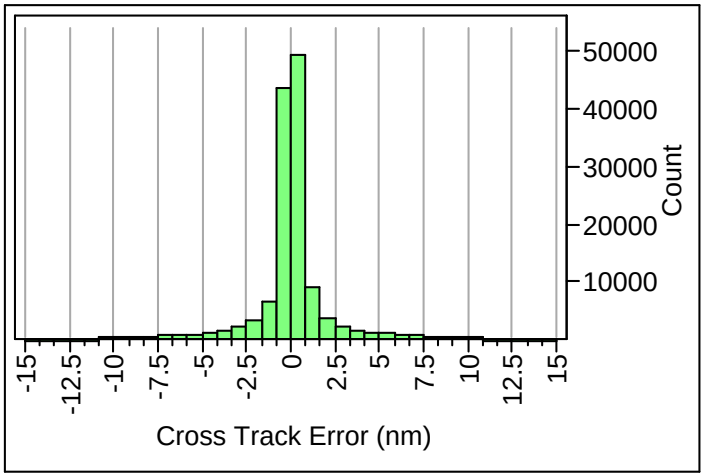
Sample Flight's Trajectory Errors per LH Time Window - Median

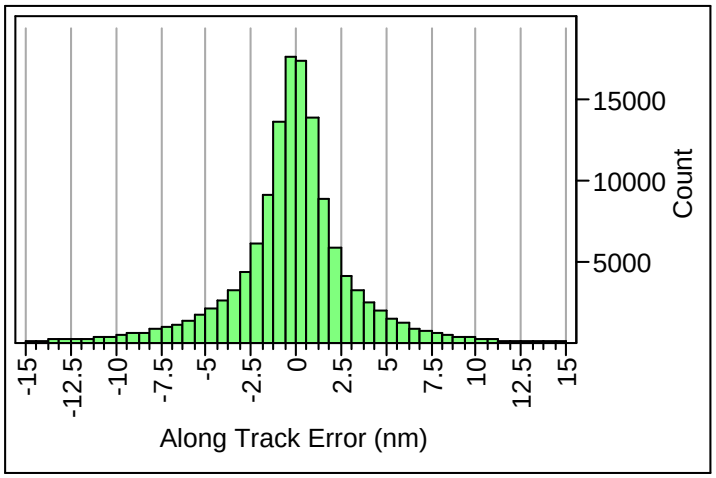


Sample Traffic Scenario of Flights

- **ZDC Traffic Sample from March 2005:**
  - 2024 flights with trajectories
  - 460K track reports and 14K clearances
- **TP produced 34K trajectories**
- **IBST**
  - With 2 min sample time and LH time of 0, 5, 10, 15, and 20 min
  - Sampled 17K trajectories
  - Calculated 140K measurements







Sample Scenario - Hypothesis Test

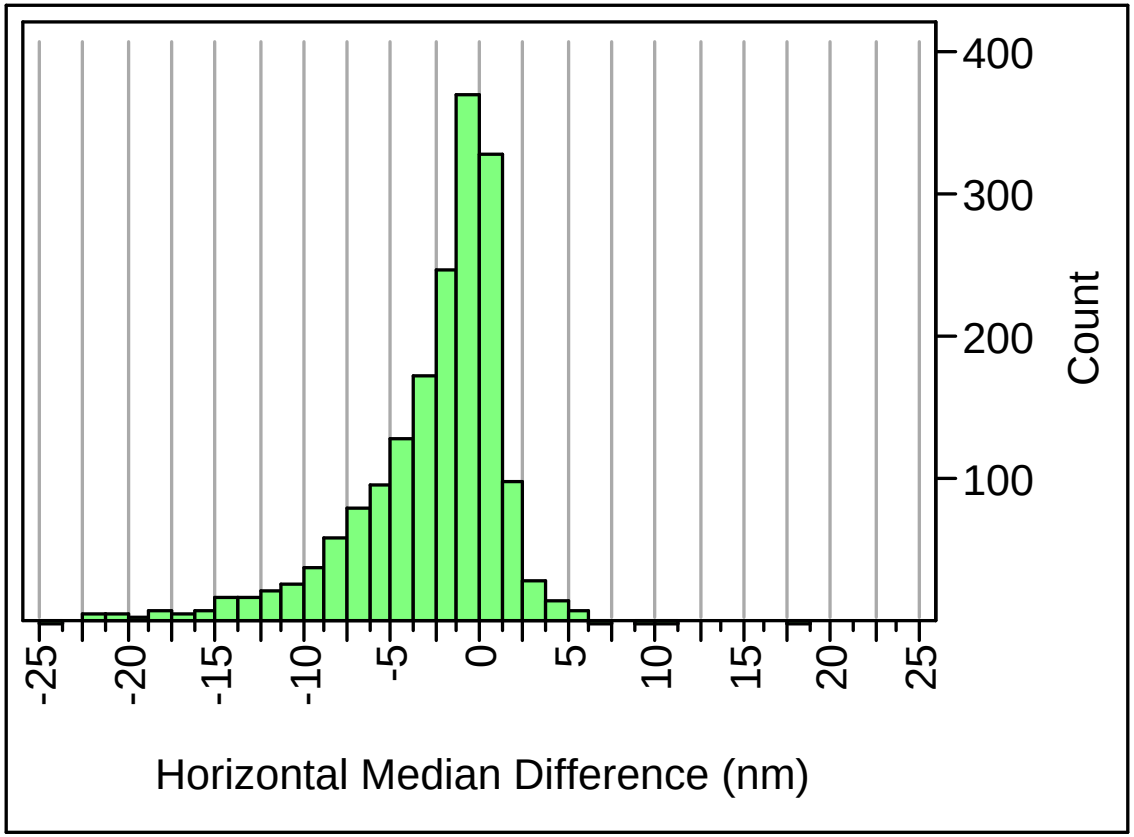
- **Some trajectories produced before surveillance data (track) was input to TP**
- **Hypothesis:**
  - Horizontal error taken before track equal to others taken after
  - Calculate median horizontal error per flight before and after track start
- **Statistic**
  - 1823 differences available
  - Mean of difference per flight, -3 nautical miles

$$t = \frac{\bar{d}}{s_D / \sqrt{n}} = \frac{-3}{5.85 / \sqrt{1823}} = -21.89$$

$$\text{Reject null hypothesis since } t \leq \left(-t_{0.05,1822} = -1.96 \right)$$

- **Thus, horizontal error is statistically significant before track start!**

Sample Scenario - Hypothesis Test Continued:



Conclusion

- **TP is critical for DST performance**
- **Trajectory accuracy methodology**
  - IBST
  - Metrics
  - Analysis
    - Facilitated by database
    - Utilize inferential statistics – pair the data!
    - Supported by specialized GUI
- **Joint Program Development Office's trajectory based operations of future NAS ensure methodology even more important!**

Mike M. Paglione
FAA Simulation & Analysis Group
mike.paglione@faa.gov
609-485-7926

Robert D. Oaks
General Dynamics Information Technology
robert.ctr.oaks@faa.gov
609-485-6175