Determination of Lateral Flight Adherence in Recorded Air Traffic Data

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When using recorded air traffic data to measure the accuracy of air traffic management decision support tools that use intent-based trajectory modelers, it is necessary to determine whether or not an aircraft is adhering to its known flight plan clearance. This paper defines what is meant by adherence and presents metrics that can be used to define lateral flight adherence. The paper describes an algorithm that is currently being applied that uses a subset of these metrics. The paper then presents a number of examples obtained from recorded air traffic data, which show instances where aircraft deviate from their known lateral clearance. The paper then presents a number of alternative approaches that could be used to implement a better algorithm for determining whether or not an aircraft was in lateral adherence based on recorded air traffic data.

I. Introduction

To achieve the goals of the Federal Aviation Administration's (FAA) 2005-2009 Flight Plan[1] for increased safety and greater capacity, advances in ground and airborne automation are required. The FAA has sponsored the development and deployment of two ground based air traffic management decision support tools (DSTs) to aid en route and terminal air traffic controllers. The User Request Evaluation Tool (URET), developed by the MITRE Center for Advanced Aviation System Development, facilitates the controller's management of en route air traffic by identifying potential aircraft-to-aircraft and aircraft-to-airspace conflicts. The Center TRACON Automation System (CTAS), developed by the NASA Ames Research Center, supports the controller in the development of arrival sequencing plans and the assignment of aircraft to runways to optimize airport capacity.

A fundamental component of both URET and CTAS is the trajectory modeler, upon which the functionality provided by these tools is based. For example, URET models aircraft trajectories to predict conflicts and CTAS models aircraft trajectories to calculate meter fix crossing times. Both URET and CTAS implement intent-based trajectory modelers. To model the aircraft's future path, the trajectory modeler inputs flight plan and clearance information contained within the Host Computer System (HCS) database. Therefore, any deviation between the known flight plan clearances and the actual path of an aircraft has a direct effect on the overall accuracy of the tool.[2-5]

In 1996 the FAA established the Conflict Probe Assessment Team (CPAT) at the William J. Hughes Technical Center to evaluate the accuracy of the conflict probes in DSTs. Since its creation CPAT has measured the conflict prediction accuracy of URET,[5] measured the trajectory modeling accuracy of both URET and CTAS,[6] conducted a study that measured the sensitivity of the URET conflict probe to weather forecast errors,[7] and assisted in the formal accuracy testing of URET Current Capability Limited Deployment (CCLD), which was the initial operational implementation of URET. CPAT is currently developing risk reduction and regression testing scenarios that are being used for the full deployment of URET into the Air Route Traffic Control Centers (ARTCC).

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As a result of these efforts, CPAT has defined four broad categories of metrics to specify the accuracy of a conflict probe: trajectory accuracy, conflict prediction accuracy, conflict notification timeliness, and conflict prediction stability. In addition, since both URET and CTAS use intent-based trajectory modelers, CPAT categorizes these metrics with respect to whether or not an aircraft was adhering to its flight plan clearances. This is important since an intent-based trajectory modeler would not be expected to do well if the aircraft is not adhering to these clearances. This adherence can be measured in both the vertical and lateral dimensions. This paper contains flight examples extracted from recorded flight data that help to understand the concept of lateral adherence and the methods associated with determining whether or not a flight is declared to be in lateral adherence.

The determination of adherence discussed in this paper is a post processing process and should not be confused with the real time problem that must be solved by trajectory modelers. This real time process is called conformance monitoring (see Ref. 8-11 for an example of detailed research on conformance monitoring).

II. Determination of Adherence

A. Definition of Adherence

In Ref. 12, CPAT established the following definition of adherence:

> The adherence to the current Air Traffic Control (ATC) clearance is defined as the status of whether the aircraft is following its known clearance at each instance of time during its flight.

Generally, adherence is considered to have two components: lateral adherence and vertical adherence. Lateral adherence refers to an aircraft's compliance with its cleared route as projected onto a stereographic x-y plane. Vertical adherence refers to whether the aircraft is maintaining, or attempting to achieve, its cleared altitude. This paper discusses lateral adherence only.

As with any definition, this definition is subject to interpretation, but CPAT has interpreted lateral adherence to mean that the surveillance track position for an aircraft should be declared out of lateral adherence when it is determined that the aircraft's intent was to deviate laterally from its known cleared route. There are many error sources for this discrepancy, but often it is caused by latencies or limitations in the data entry of these clearances into the automation systems. In Ref. 13, MITRE researchers reported that only about 30 percent of the lateral clearances were manually entered into the automation system by air traffic controllers. The result is significant error in the DST's aircraft trajectory prediction. In another study reported in Ref. 14, FAA CPAT researchers found 90 percent of the surveillance track positions of a 1999 five-hour traffic sample had lateral deviations as high as 9 nautical miles and this grew substantially to 19 nautical miles by the 97.5 percentile of the data sample.

B. Geometry of Lateral Adherence

Figure 1 shows the geometry associated with determining lateral adherence of a surveillance track data point for an aircraft. The figure shows an aircraft at a specific position and flying along a path in a specific direction. The figure also shows the current route segment for the aircraft with a triangle depicting the next fix on this route. Identified in this figure are five metrics that can be used to define whether or not an aircraft is in lateral adherence at a specific data point. These five metrics are:

1. α - The angle between a line drawn from the aircraft's track data point to the next fix to and a line coincident with the aircraft's current route segment.
2. β - The angle between the aircraft's heading vector and a line drawn from the aircraft's track data point to the next fix.
3. d_a - The distance along the route to the next fix measured from a point normally projected from the aircraft's track data point onto the aircraft's current route segment.
4. d_n - The straight line distance between the aircraft's track data point and the next fix.
5. d_r - The normal distance from the aircraft's track data point to the aircraft's current route segment.

Based on this geometry, a track data point for an aircraft would be considered to be in "perfect lateral adherence" when all of the following four conditions are true:

$$\alpha = 0 \tag{1}$$

$$\beta = 0 \tag{2}$$

$$d_r = 0 \tag{3}$$

$$d_a = d_n \tag{4}$$

In this state of "perfect lateral adherence," the aircraft's track data point would be on the current route segment and track flight path heading directly toward the next fix with the metrics $d_a = d_n$ indicating the distance to the next fix. It is unlikely this state would ever exist, but these metrics can be used algorithmically to determine whether or not an aircraft's tack data point is in lateral adherence.

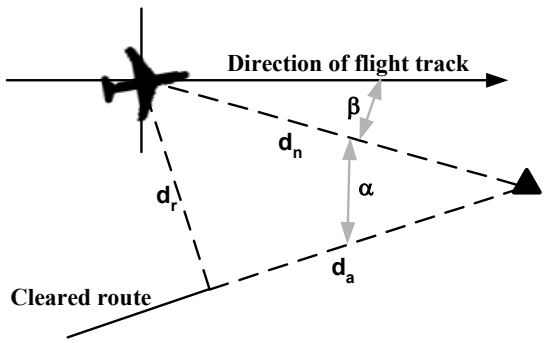

Figure 1 The Geometry of Lateral Adherence

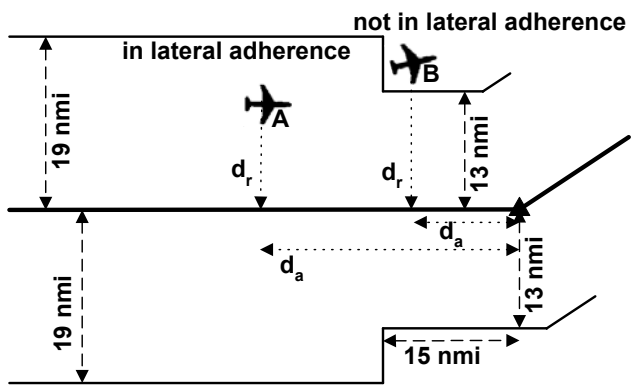

Figure 2. CPAT Lateral Adherence Determination Algorithm Parameters and Thresholds

C. The CPAT Lateral Adherence Determination Algorithm

The CPAT Lateral Adherence Determination Algorithm is an example of an algorithm that determines whether or not an aircraft's track data point is in lateral adherence. This algorithm uses only two of the geometry of lateral adherence metrics: d_r and d_a. The algorithm declares an aircraft's track data point to be in or out of adherence depending whether the d_r is less than or greater than a threshold value. This threshold value is not a constant; a smaller threshold is used when the d_a for the aircraft track data point is within a parametric distance to a turn fix, where a turn fix is defined as a fix for which the route heading change exceeds 30 degrees. These threshold values and the parametric distance vary by altitude. Ref. 14 describes the process CPAT used to establish these thresholds and parametric values.

For the examples presented in this paper the threshold values and parametric distance established in Ref. 14. for aircraft flying at or between altitudes of 18000 and 33000 feet were used. This is shown graphically in Figure 2. In this figure, the aircraft labeled A is declared to be in lateral adherence because its d_a is greater than 15 nmi and its d_r is less than 19 nmi. The aircraft labeled B is declared not to be in lateral adherence because its d_a is less than 15 nmi and its d_r is greater than 13 nmi.

III. Examples

The data for the following examples was extracted from air traffic data recorded in the Indianapolis Air Route Traffic Control Center (ZID). The aircraft identifiers have been changed so that specific flights cannot be identified. In the figures accompanying these examples the recorded track data points of the aircraft are shown as a solid line and the aircraft's converted route (i.e., its route based on its flight plan) is shown as a dashed line. The triangles represent waypoints along the route. An arrow showing the general direction of flight is also shown on each of the figures. Where the aircraft track data points are black the CPAT Lateral Adherence Determination Algorithm found the track data point to be in adherence; where the track data points are a lighter gray the algorithm found the track data point to be out of adherence (i.e., beyond the specified thresholds defined in Ref. 14).

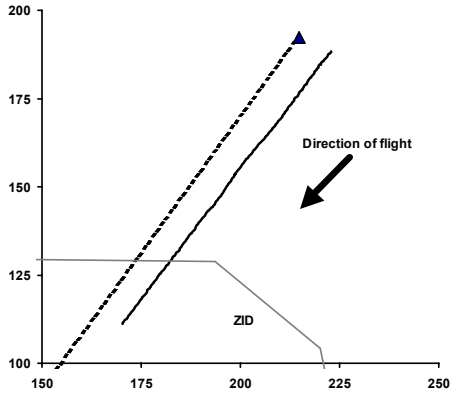

Figure 3. ABC101 Track and Route

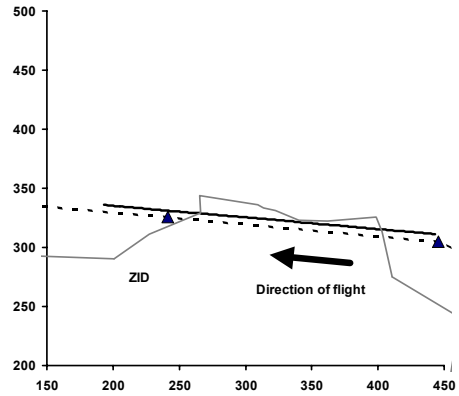

Figure 4. ABC102 Track and Route

A. Example 1

Based on the CPAT Lateral Adherence Algorithm presented in Section II, an aircraft's track position will not necessarily be coincident with its filed converted route to be declared in lateral adherence; nor does it have to fly over the waypoints along its route. Figures 3 and 4 show examples of these situations. Figure 3 shows ABC101 flying in a southwesterly direction approximately 8 nmi to the left of its route. Figure 4 shows ABC102 flying in a westerly direction approximately 4 nmi to the right of its route. For both of these cases the CPAT Lateral Adherence Determination Algorithm declared all track data points for these two flights to be in adherence.

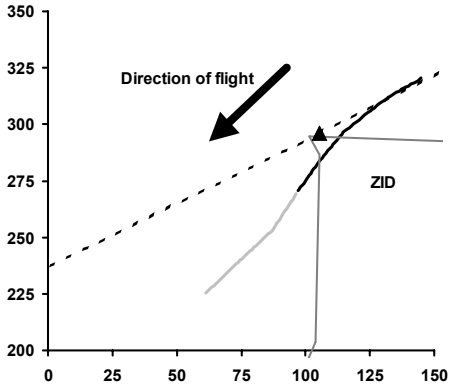

Figure 5. ABC201 Track and Route (Close Up)

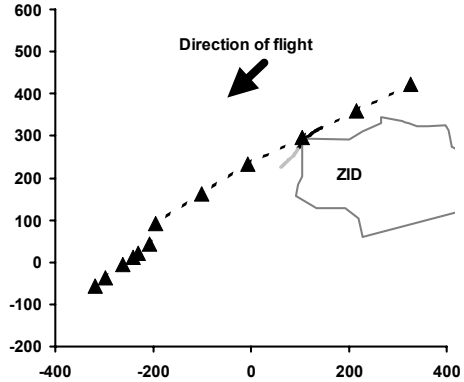

Figure 6. ABC201 Track and Route (Wide Area)

B. Example 2

Figures 5 and 6 show situations for which the CPAT Lateral Adherence Determination Algorithm declared an aircraft's track data points to be out of adherence. Figure 5 shows that ABC201 flew in a southwesterly direction coincident with its cleared route; then prior to crossing one of the waypoints the aircraft turned to its left. The gray

line indicates the track data points at which the CPAT Lateral Adherence Determination Algorithm declared the aircraft to be out of adherence (i.e., these track data points are more than 19 nmi from the cleared route). Figure 6 presents a wider view of this same situation, which shows that it is likely that an air traffic controller had advised the pilot to fly direct to one of the future fixes on the aircraft's route, but that this information was not entered into the HCS database.

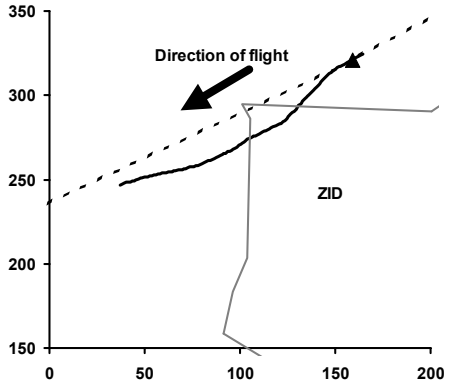

Figure 7. ABC301 Track and Route (Close Up)

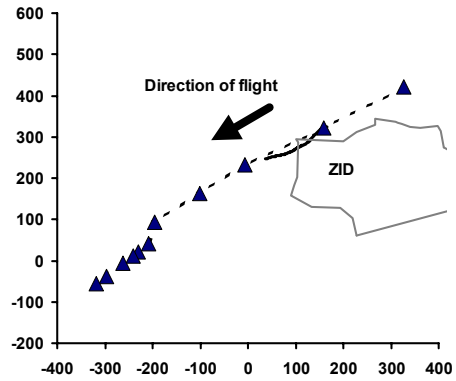

Figure 8. ABC301 Track and Route (Wide Area)

C. Example 3

Figures 7 and 8 show data for ABC301, which had the same cleared route as ABC201 presented in Example 2. It appears that an air traffic controller gave the same direct to fix advisory to ABC301, but then provided another advisory returning ABC301 to its original route. At its furthest point, ABC301 was about 17.5 nmi away from its cleared route. The CPAT Lateral Adherence Determination Algorithm found no points out of adherence for this flight since no lateral distances exceeded the threshold of 19 nmi.

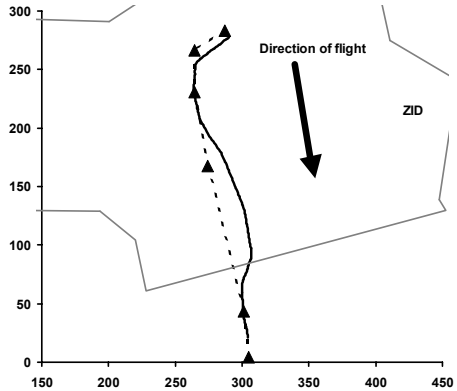

Figure 9. ABC401 Track and Route

D. Example 4

Figure 9 shows that ABC401 flew in a southwesterly direction slightly to the left of its cleared route. It then turned in a southerly direction and merged with its route. After flying over a fix, ABC401 turned to its left diverging from its route and bypassed one of its waypoints, probably because an air traffic controller advised the pilot to alter its route. Later the aircraft turned to its right and merged with its cleared route. The CPAT Lateral Adherence Determination Algorithm declared this aircraft to be in adherence for all of its track data points.

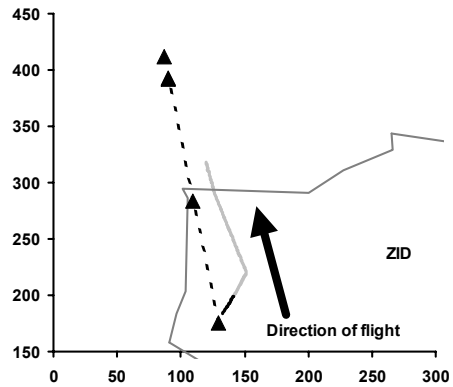

Figure 10. ABC501 Track and Route

E. Example 5

Figure 10 presents the data for a departure aircraft ABC501. It is likely that this aircraft departed on a different runway than it had originally planned. As a result the aircraft climbed in a northeasterly direction rather than the north-northwesterly direction shown in its cleared route. Once the aircraft achieved an altitude of 14000 feet, it turned to the northwest and flew toward one of the fixes on its route. The CPAT Lateral Adherence Determination Algorithm declared the aircraft to be in adherence during its ascent until it reached the threshold distance from the route, and then declared the aircraft to be out of adherence for the remainder of the data. Assuming the aircraft ultimately merged with its route, the CPAT Lateral Adherence Determination Algorithm would have declared the aircraft to be in adherence as soon as the distance to the route became less than the threshold value.

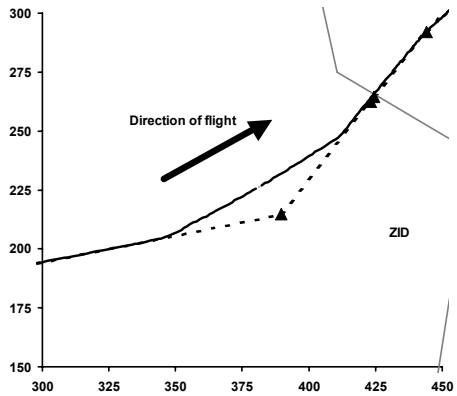

Figure 11. ABC601 Track and Route

F. Example 6

Figure 11 shows a case where an aircraft "cuts a corner." The data in this figure shows that ABC601 flew on its route in an east northeasterly direction toward a fix, then turned left bypassing the fix, and then merged with the next route segment. The CPAT Lateral Adherence Determination Algorithm called one point out of adherence for this flight.

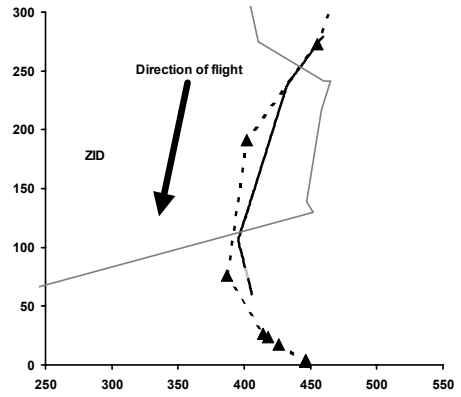

Figure 12. ABC701 Track and Route

G. Example 7

Figure 12 presents an interesting situation of consecutive corner cutting. The data in this figure shows that ABC701 flew in a south southwesterly direction coincident with its cleared route. About half way between two fixes, it turned to its left bypassing its next fix and flew directly to a future fix. As it neared that fix, it again turned to its left bypassing that fix and flew directly toward another future fix. The CPAT Lateral Adherence Determination Algorithm declared only a small segment of this flight to be out of adherence. This segment is shown by the gray track that represents the short period of time around a turn fix.

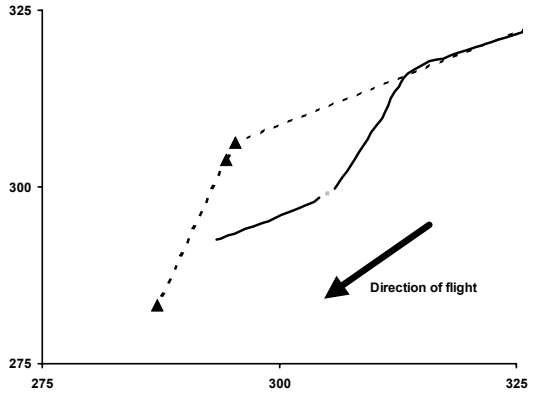

Figure 13. ABC801 Track and Route

H. Example 8

Figure 13 shows another example of what may be considered "cutting a corner." The data in this figure shows that ABC801 flew along its cleared route in a southwesterly direction, then turned to its left bypassing two fixes, then began to fly back to a route segment. What makes this example different is that the aircraft did not appear to be flying directly toward any fix as it bypassed fixes. The CPAT Lateral Adherence Determination Algorithm declared one data point to be out of adherence for this flight.

IV. Alternative Lateral Adherence Algorithms

These examples show that it is not uncommon for aircraft to deviate laterally from their cleared routes known to the DST. They also show that the CPAT Lateral Adherence Determination Algorithm functions correctly with regards to its design; however in many cases the algorithm gives results that could be considered incorrect. Specifically, with regards to the preceding examples:

- It is probably correct that both ABC101 and ABC102 in Example 1 were declared to be in adherence. But at what distance should an aircraft flying parallel to its route be declared to be out of adherence?
- ABC201, in Example 2, should probably be declared to be out of adherence sooner.
- ABC301 and ABC401, in Examples 3 and 4, should probably be declared to be out of adherence as they deviated from its route, and then declared to be in adherence when they turned and began flying toward their routes.
- ABC501, in Example 5, should probably be declared to be out of adherence as it departed, then declared to be in adherence when it turned and began flying toward its route.
- ABC601, ABC701, and ABC801, in Examples 6, 7, and 8, should probably be declared out of adherence when they turned off their cleared route segment and were no longer flying toward their next fix.

A. Enhancement of the Current Algorithm

One approach to improving the current CPAT Lateral Adherence Determination Algorithm would be to enhance its current capabilities by considering more parameters. As stated previously the current algorithm uses only two of the five parameters defined in the Geometry Of Lateral Adherence presented in Figure 1: the lateral distance to the current line segment (d_r) and the distance along the current line segment to the next node (d_a). One simple enhancement that could easily be added to the current algorithm would be to establish a threshold on the bearing to the next node (β). This could resolve problem that can occur in situations such as the one depicted graphically in Figure 14. In this situation the aircraft labeled C would be declared to be in adherence even though it is flying away from its route, which is problem that occurs in many of the preceding examples.

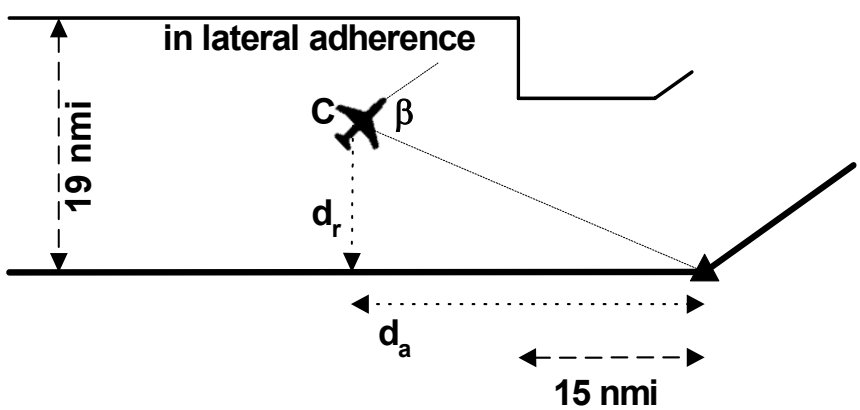

Figure 14. Example Showing Problem With Not Using β

B. Implementation of a Conformance Monitoring Technique

Another approach would be to implement a Conformance Monitoring technique using Fault Detection logic. This technique was referenced earlier in this paper as an example of a detailed description on conformance monitoring.[8-11] Much of this research is consolidated in Ref. 15, in which a Conformance Monitoring Analysis Framework is presented that provides a structure to describe and research conformance monitoring approaches. The information in this paper could be implemented as a Lateral Adherence Determination Algorithm.

C. Implementation of a Statistical Quality Control Technique

Another approach would be to implement a Statistical Quality Control (SQC) technique in a manner similar to that proposed in Ref. 16, in which an X-Bar Control Chart was applied as a method for detecting when an aircraft deviates laterally from its known cleared route. SQC is used extensively in manufacturing processes where key system parameters are monitored in real time for conformance to established probability distributions. If the parameter exceeds certain limits, for example set to three standard deviations of the mean value, it is declared out of control. This is analogous to aircraft deviating from its known flight plan. In the sample data provided in Ref. 16, the X-Bar Control Chart was shown to be more effective than the use of a constant threshold when using only the lateral distance to the route (d_r). It could perhaps be even more effective if other parameters were monitored; especially the bearing to the next node (β).

D. Development of an Algorithm Based on Fuzzy Set Theory

Another approach that could be considered is the use of fuzzy set theory, which is an emerging technology that is successfully being applied to many information technology problems. Numerous books are available (such as Ref. 17) that provide detailed information about this technology. To understand how fuzzy set theory could be applied to determining lateral adherence, consider that S represents a set with a membership function denoted $S(x)$. In normal set theory this membership function would be of the form:

$$S(x) = \begin{cases} 1, & if\ x \in S \\ 0, & if\ x \notin S \end{cases} \tag{5}$$

where a membership value of 1 would indicate membership in the set a value of 0 would indicate non-membership. In fuzzy set theory the membership function would be of the form:

$$S(x) = [0.0, 1.0] \quad for\ \forall\ x \tag{6}$$

where membership can assume real values between 0.0 and 1.0 representing a degree of membership.

As an example of how this could be applied to determining lateral adherence consider two fuzzy sets A and B based on two angular parameters defined previously in the Geometry of Lateral Adherence that identify when an aircraft is out of adherence: (1) the angle off route (α), which is another way of expressing an aircraft's lateral deviation from the route, and (2) β, which is a measure of whether the aircraft is flying toward its next fix.

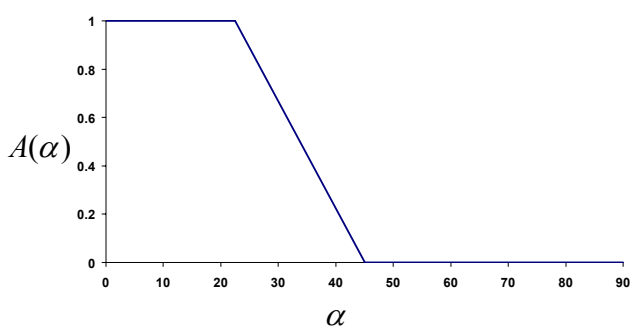

Figure 15. Membership Function for α

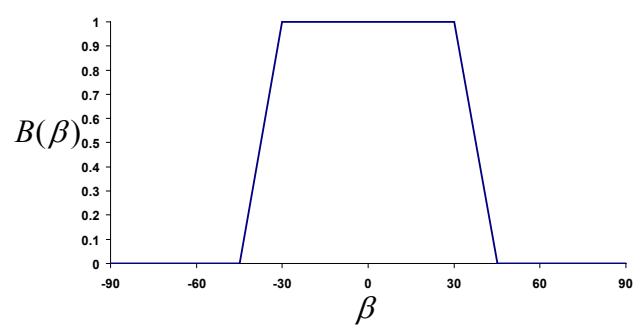

Figure 16. Membership Function for β

As an example, consider Figure 15 to represent the membership function for the set A. This represents that if the angle off route is less than 22.5 degrees ($\alpha < 22.5°$) the aircraft would be considered to be in adherence (membership in A of 1.0) and if the angle off route is greater than 45 degrees ($\alpha > 45°$), the aircraft would be considered to be out of adherence (membership in A of 0.0). The membership value for angles between 22.5 degrees and 45 degrees can be determined through interpolation.

Also consider Figure 16 to represent a membership function for the set B. This represents that if the bearing to the next fix is less than a negative 45 degrees ($\beta < -45°$) or greater than a positive 45 degrees ($\beta > 45°$) the aircraft would be considered to be out of adherence (membership in B of 0.0) and if the bearing to the next fix is between a

negative 30 degrees and a positive 30 degrees (-30º < β < 30º) the aircraft would be considered to be in adherence (membership in B of 1.0). The membership value for angles between -45 degrees and -30 degrees and between 30 degrees and 45 degrees can be determined through interpolation.

As in normal set theory, operations such as intersection ($\cap$) and union ($\cup$) are defined for fuzzy sets. In particular another fuzzy set denoted C, representing adherence, could be defined as the union of the sets A and B. Specifically, without going into details this would be represented as:

$$C(\alpha,\beta) = A(\alpha) \cup B(\beta) = \max(A(\alpha), B(\beta)) \tag{7}$$

For example, if at a track point α = 30º, then $A(\alpha)$ = 0.667; and if β = 35º, then $B(\beta)$ = 0.333. Then the adherence at that track point would be 0.667. Note that as a consequence of using fuzzy set theory, adherence would become a fuzzy variable assuming values between 0.0 and 1.0, representing degrees of adherence.

E. Development of an Algorithm that uses Multiple Track Points

Another approach would be to use multiple track points, rather than considering each point independently, as applied in each of the preceding alternatives. Such an algorithm could consider a number of points before and after the track point under consideration. Techniques that could be used include the computation of a statistical correlation coefficient or of root-mean-square (RMS) values using the track data points and interpolated points along the cleared route. One problem that would need to be resolved for the use of this approach would be how to match the track data points with points along the cleared route since there is no time associated with the cleared route.

V. Conclusion

In order to properly measure the accuracy of an air traffic management decision support tool that uses an intent-based trajectory modeler, it is necessary to identify the track data at which an aircraft is not laterally adhering to its known cleared flight plan. This assures the decision support tool is not unjustly penalized for poor predictions or at minimum the source of the error is identified. The Lateral Adherence Determination Algorithm currently used by CPAT does not function adequately; therefore another algorithm should be developed. Five potential approaches identified in this paper are:

1. Enhancement of the current algorithm.
2. Implementation of a conformance monitoring technique.
3. Implementation of a Statistical Quality Control technique.
4. Development of an algorithm based on fuzzy set theory.
5. Development of an algorithm that uses multiple track points.

Each of these approaches is viable and it would require a thorough analysis, using recorded air traffic data, to determine which of these approaches should be implemented.

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