MIT International Center for Air Transportation

A Two-Stage Optimization Methodology for Runway Operations Planning

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Joint University Program,
Cambridge, Massachusetts
October 17-18 , 2002

Surface Operations Optimization: Problem Structure

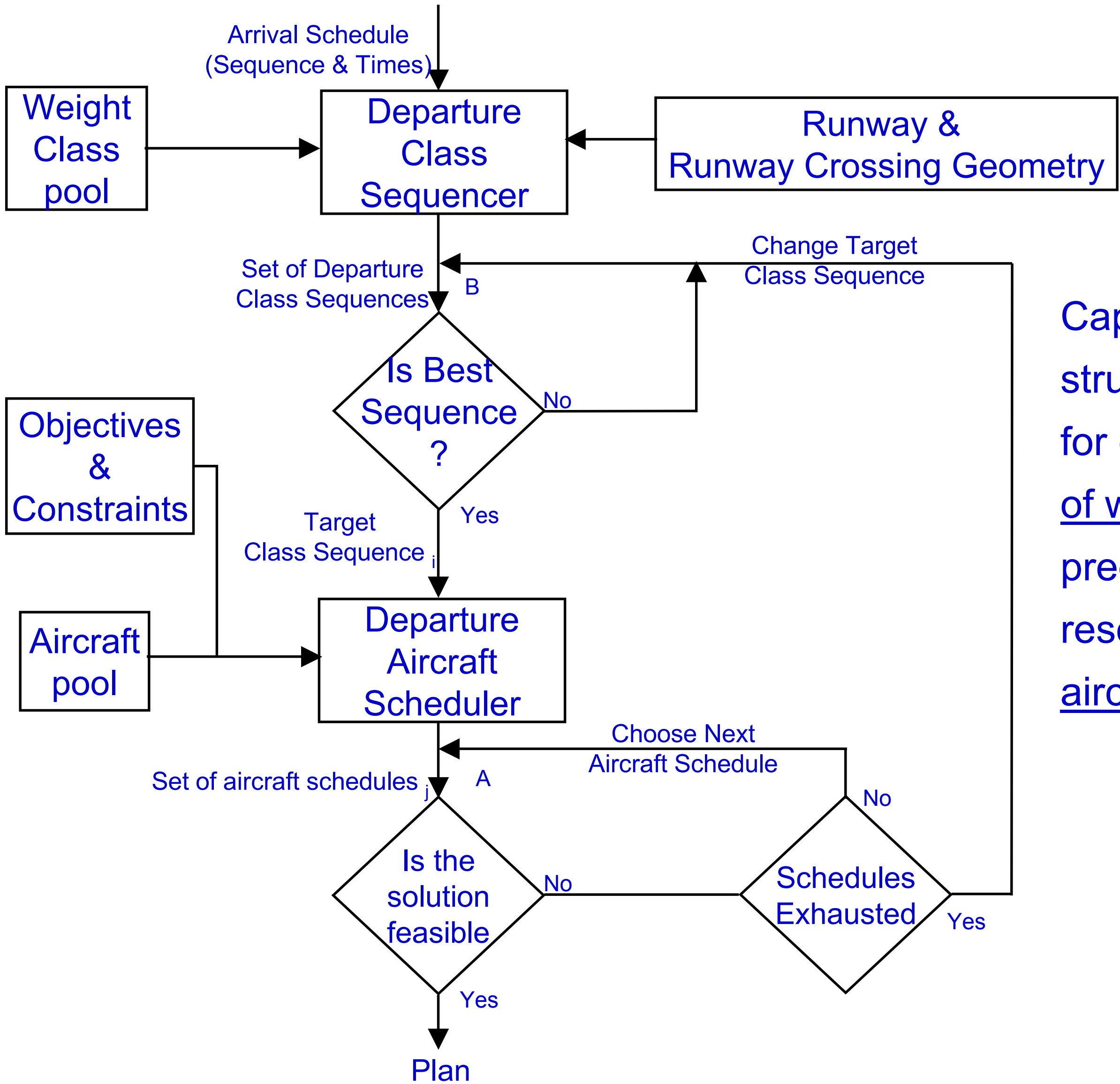
- Surface Operations Optimization (SOO)
  - Surface Operations Planning (SOP)
    - Runway Operations Planning (ROP)
    - Taxiway – Gate Operations Planning (TGOP)
  - Surface Operations Control (SOC)

- SOP: Develop feasible and optimal departure plans that achieve desired objectives
- SOC: Execute plans

Two Stage
Runway Operations Planning

- Stage 1: Departure Class Scheduling
  - Assume fixed arrival schedule and resulting crossing requests on the departure runway
  - Generate departure class schedules with crossings based on a SINGLE objective (max. expected throughput) and the runway geometry
  - Matrix of class schedules CS = {CS1, … , CSi, … , CSm}
- Stage 2: Departure Aircraft Scheduling
  - For a Target Class Sequence optimize other objectives by assigning specific flights of the pre-assigned weight class for each departure class slot.
  - Matrix of aircraft schedules AS = {AS1, … , ASj, … , ASn}

Two Stage Runway Operations Planning



Capitalize on the problem structure I.e. predicted demand for departure resources <u>in terms of weight class</u> more certain than predicted demand for departure resources <u>in terms of specific aircraft</u>

Stage 1
Solution Approach

- Calculate throughput (with crossings) for each sequence
- Rank sequences based on throughput with crossings
- Use historical data to determine the probability that position shifts will occur
- Calculate “expected” throughput given potential position shifts
- Rank sequences based on “expected” throughput
- Select best sequence as most robust solution

Stage 1: Expected Throughput Example

- Base Class Schedule with possible swaps (X1 and X2 are crossing groups)

L – L – L – H – X1 – H – X2 – L – L – L – L – L – L – S – S – S

- Class Schedules derived from the Base Schedule

L – L – L – H – x1 – H – x2 – L – L – L – L – L – L – S – S – S
L – L – L – H – x1 – H – x2 – L – L – L – L – L – **S** – **L** – S – S
L – L – L – H – x1 – **L** – x2 – **H** – L – L – L – L – L – S – S – S
L – L – L – H – x1 – **L** – x2 – **H** – L – L – L – L – **S** – **L** – S – S
L – L – **H** – **L** – x1 – H – x2 – L – L – L – L – L – L – S – S – S
L – L – **H** – **L** – x1 – H – x2 – L – L – L – L – L – **S** – **L** – S – S
L – L – **H** – **L** – x1 – **L** – x2 – **H** – L – L – L – L – L – S – S – S
L – L – **H** – **L** – x1 – **L** – x2 – **H** – L – L – L – L – **S** – **L** – S – S

- Throughput and probability of occurrence calculated for each “derived” schedule
- To increase schedule robustness to unforeseen events, stochastic throughput for “base” schedule calculated as the expected value over all “derived” schedules

Stage 2
Solution Approach

- Assignment Problem (undirected graph with aircraft and slot nodes)
- Decision Var. X_{ij} (graph link (i,j)) = 1 if aircraft i populates slot j, otherwise 0
- Stage 2 objective function minimizes departure or aggregate delay (time - based)
  - Cost of link (i,j) is the delay aircraft i absorbs due to this assignment
- Other constraints are introduced only if binding
  - If there are no violations, solution accepted, otherwise …
  - Assignment of one of the violating aircraft is prevented by setting assignment cost to a very large value and re-solve the assignment problem

Objective Function

- Delay based

  - Min aggregate delay

$$q(x) = \sum_{i=1}^{N_D} \left| TOff_i(x_i) - EOff_i \right|^{k_D} + \sum_{j=1}^{N_A} \left| TOn_j - EOn_j \right|^{k_A} + \sum_{m=1}^{N_A} \left| TX_m - EX_m \right|^{k_X}$$

  - Min departure delay

$$q(x) = \min \sum_{i=1}^{N_D} \left| TOff_i(x_i) - EOff_i \right|^{k_D}$$

with $1 \le i \le N_D$ and $1 \le j,m \le N_A$

where: N_A is the total # of arrivals, N_D is the total # of departures, and k_A, k_D and k_X are parameters used to penalize delays of specific flights $(k_A \ge 1, k_D \ge 1, k_X \ge 1)$

Stage 2 Constraints

- Earliest Time Possible rules certain slots out for certain aircraft
- Class Slot Restrictions: Stage 1 “class to slot” assignments to be obeyed
  - Example: aircraft 1 is the only non-Large aircraft, only Large slots are slot 2,3,4,5 and 6

$$\mathrm{X}_{1\mathrm{j}} = 0,\ \forall\ \mathrm{j} \in [2, 3, 4, 5, 6] \quad \text{or} \quad \sum_j X_{1j} = 1,\ \forall\ slot\ \ j \notin [2,3,4,5,6]$$

- Each aircraft occupies ONLY ONE slot: $\sum_{j=1}^{N_S} X_{ij} = 1,\ \forall\ aircraft\ \ i$
- Each slot is occupied by ONLY ONE slot: $\sum_{i=1}^{N_D} X_{ij} = 1,\ \forall\ slot\ \ j$
- Fairness / Deviation from FCFS / Maximum takeoff Position Shifting (MPS)
  - Pushback positions and MPS value known $\left|\mathrm{X}_{\mathrm{PB_i}} - \mathrm{X}_{TO_i}\right| \leq \mathrm{MPS} \Leftrightarrow \begin{cases} -\mathrm{X}_{TO_i} \leq \mathrm{MPS} - \mathrm{X}_{\mathrm{PB_i}} \\ \mathrm{X}_{TO_i} \leq \mathrm{MPS} + \mathrm{X}_{\mathrm{PB_i}} \end{cases} \forall\ \text{aircraft}\ \ \mathrm{i}$
  - Aircraft takeoff position can be written as: $X_{TO_i} = \sum_{j=1}^{N_S} j * X_{ij}$
  - Therefore:

$$\begin{cases} -\sum_{j=1}^{N_S} j * X_{ij} \leq \mathrm{MPS} - \mathrm{X}_{\mathrm{PB_i}} \\ \sum_{j=1}^{N_S} j * X_{ij} \leq \mathrm{MPS} + \mathrm{X}_{\mathrm{PB_i}} \end{cases} \forall\ \text{aircraft}\ \ \mathrm{i}$$

Stage 2 Constraints

- Time window constraints (EDCT, DSP) translated to fixed position constraints

  - EDCT: $\mathrm{EDCT}_{i_1} \le \sum_{j=1}^{N_S} j * X_{ij} \le \mathrm{EDCT}_{i_2}$

  - DSP: $\mathrm{DSP}_{i_1} \le \sum_{j=1}^{N_S} j * X_{ij} \le \mathrm{DSP}_{i_2}$

- Lifeguard / Special priority flights

  - Upper bound: $\sum_{j=1}^{N_S} j * X_{ij} \le \mathrm{X}_{\mathrm{maxTO}_i}$, *where* $\mathrm{X}_{\mathrm{maxTO}_i}$ *is slot upper bound*

  - Inequality between flights: $\sum_{j=1}^{N_S} j * X_{ij} \le \sum_{j=1}^{N_S} j * X_{kj} \Leftrightarrow \sum_{j=1}^{N_S} j * (X_{ij} - X_{kj}) \le 0$

- In Trail Constraints (MIT or MinT) expressed in terms of minimum takeoff position separation requirements:

$$\left| \sum_{j=1}^{N_S} j * X_{ij} - \sum_{j=1}^{N_S} j * X_{kj} \right| \ge \Delta X_{ik} \Leftrightarrow \left| \sum_{j=1}^{N_S} j * (X_{ij} - X_{kj}) \right| \ge \Delta X_{ik} \Leftrightarrow \begin{cases} \sum_{j=1}^{N_S} j * (X_{ij} - X_{kj}) \ge \Delta X_{ik} \\ \sum_{j=1}^{N_S} j * (-X_{ij} + X_{kj}) \ge \Delta X_{ik} \end{cases}$$

Sample output: 9 aircraft, No MPS

- Class Slot Sequence: S s s s S S s S L L L L L
  - At time 06:33 departure AAL1317 (S 1) takes off in slot 1 with a maximum delay of 1 minutes
  - At time 06:34 the following arrival(s) cross together:
    - S arrival N65MJ (number 1 in the arrival schedule) with a delay of 113 seconds
    - S arrival USC422 (number 2 in the arrival schedule) with a delay of 10 seconds
    - S arrival FDX1464 (number 3 in the arrival schedule) with a delay of 0 seconds
  - At time 06:38 departure N180M (S 5) takes off in slot 2 with a maximum delay of 1 minutes
  - At time 06:39 departure USC168 (S 4) takes off in slot 3 with a maximum delay of 2 minutes
  - At time 06:40 the following arrival(s) cross together:
    - S arrival ASH5268 (number 4 in the arrival schedule) with a delay of 109 seconds
  - At time 06:41 departure N109FX (S 3) takes off in slot 4 with a maximum delay of 3 minutes
  - **At time 06:42 departure SGR501 (L 9) takes off in slot 5 with a maximum delay of 2 minutes**
  - **At time 06:43 departure USA1854 (L 7) takes off in slot 6 with a maximum delay of 3 minutes**
  - **At time 06:44 departure USS6171 (L 8) takes off in slot 7 with a maximum delay of 4 minutes**
  - **At time 06:45 departure COA339 (L 6) takes off in slot 8 with a maximum delay of 5 minutes**
  - **At time 06:46 departure DAL1821 (L 2) takes off in slot 9 with a maximum delay of 9 minutes**
- Total time to complete departures is 13 minutes
- Total departure delay is AT MAXIMUM 28 minutes

Sample output: 9 aircraft, MPS = 3

- Class Slot Sequence: S s s s S S s S L L L L L
  - At time 06:33 departure AAL1317 (S 1) takes off in slot 1 with a maximum delay of 1 minutes
  - At time 06:34 the following arrival(s) cross together:
    - S arrival N65MJ (number 1 in the arrival schedule) with a delay of 113 seconds
    - S arrival USC422 (number 2 in the arrival schedule) with a delay of 10 seconds
    - S arrival FDX1464 (number 3 in the arrival schedule) with a delay of 0 seconds
  - At time 06:38 departure N180M (S 5) takes off in slot 2 with a maximum delay of 1 minutes
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  - At time 06:40 the following arrival(s) cross together:
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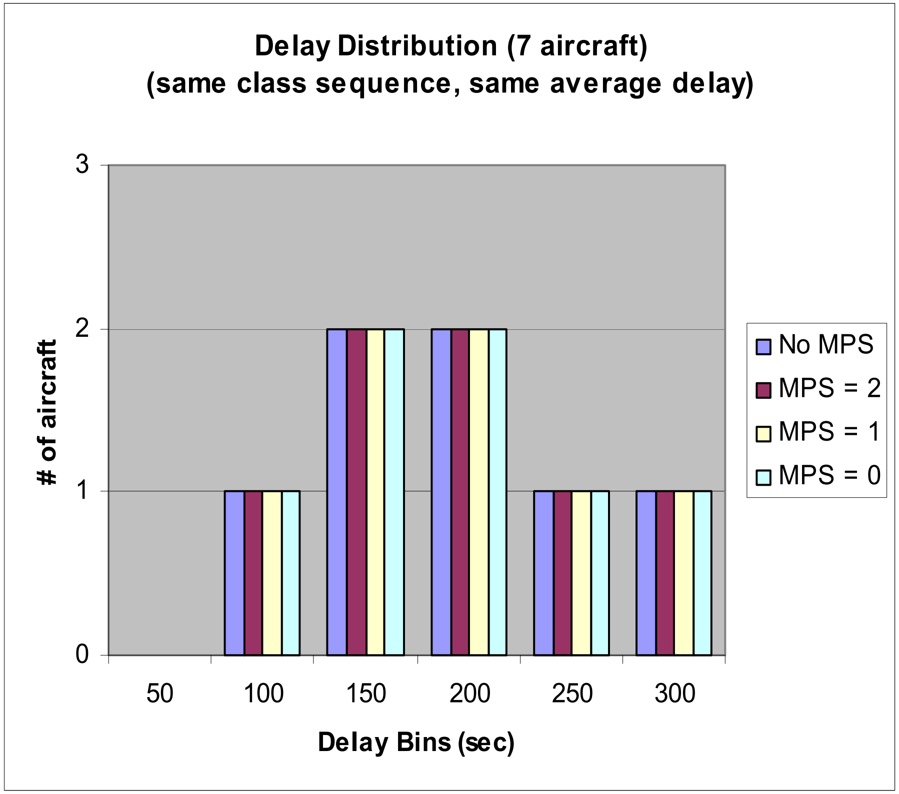
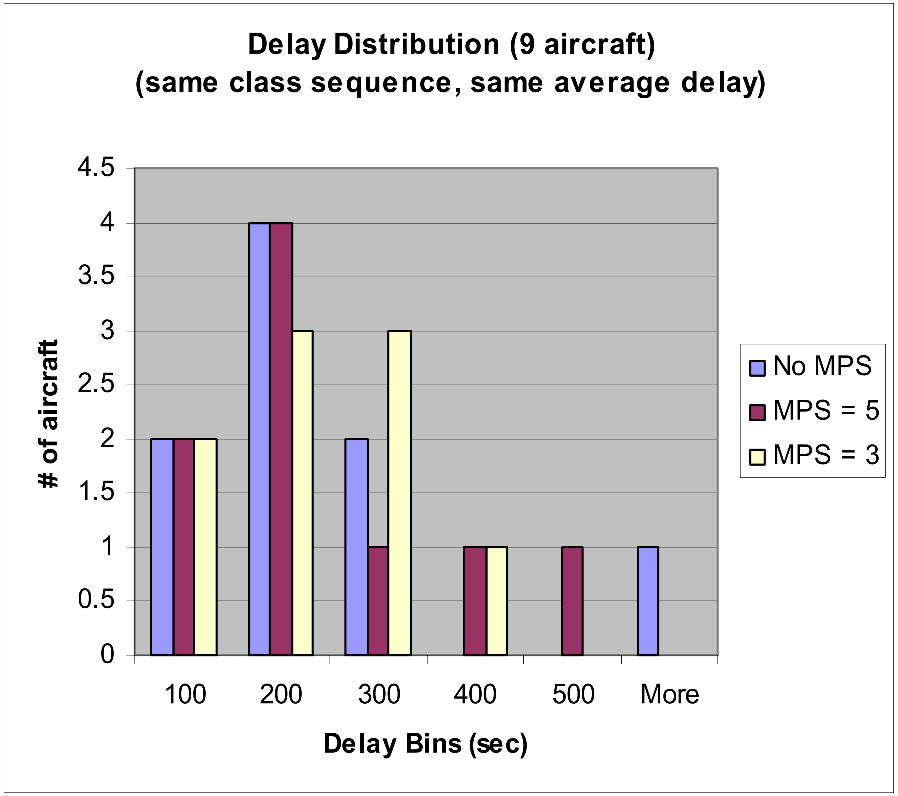
  - **At time 06:46 departure COA339 (L 6) takes off in slot 9 with a maximum delay of 6 minutes**
- Total time to complete departures is 13 minutes
- Total departure delay is AT MAXIMUM 28 minutes

Departure delay distribution

- The introduction of MPS constraints may or may not reduce segregation of delays (penalizing specific flights)





Departure slot assignments

- Departure slot assignments and therefore delay distribution (i.e. which aircraft will be delayed and how long) may depend on the weight class composition of the best (target) class schedule chosen at the end of Stage 1
  - 9 aircraft example: **only the 5 Large** aircraft change assignments from case to case
  - 7 aircraft example: **all aircraft** change assignments from case to case
- Fairness is achieved with MPS constraints, even though total delay may not change