Search Procedures For Use With Aerodynamic Shape Optimization

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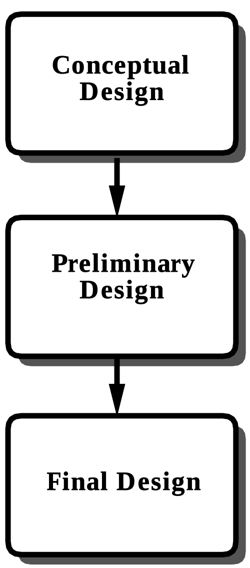
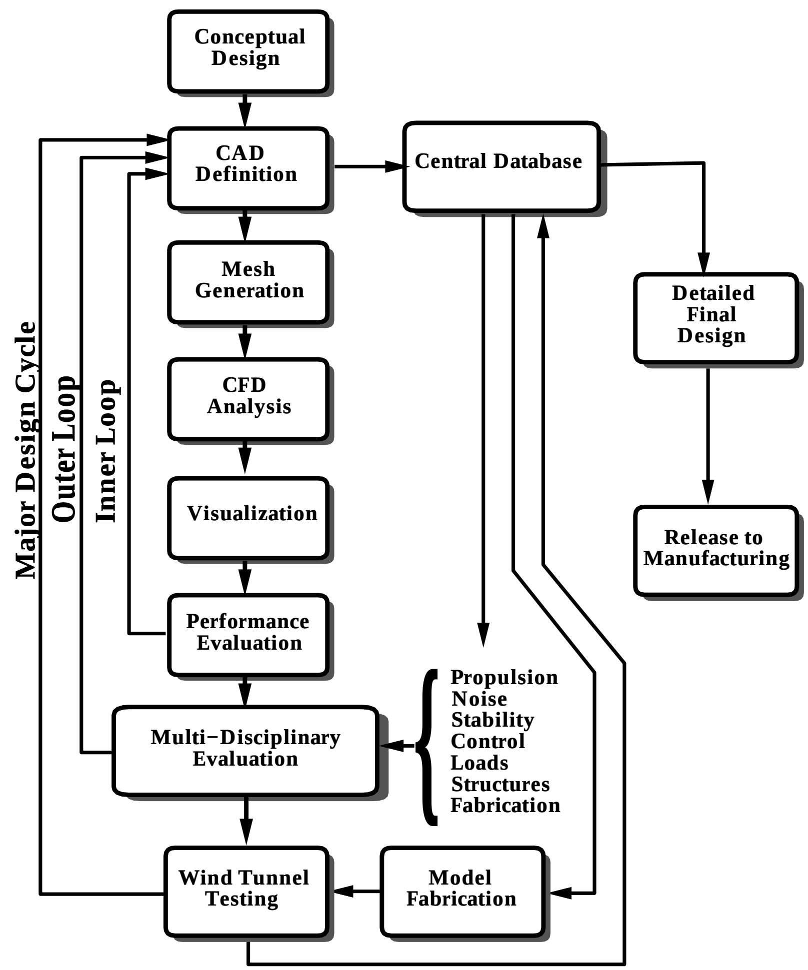
DESIGN PROCESS



Defines Mission
Preliminary sizing
Weight, performance



REQUIREMENTS FOR SIMULATION-BASED DESIGN

- Sufficient Level of Accuracy
- Known Level of Accuracy
- Acceptable Computational Costs
- Acceptable Man-Power Costs
- Fast Turn-Around Time

SHAPE DESIGN VIA CONTROL THEORY

- Apply the theory of control of PDEs (of the flow) by boundary control (the shape).

- Find the Frechet derivative (infinite dimensional gradient) of a cost function (performance measure) with respect to the shape by solving the adjoint equation in addition to the flow equation.

- Modify the shape in the sense defined by the smoothed gradient.

- Repeat until the performance value approaches an optimum.

PROCESS OVERVIEW

1. Solve the flow equations for ρ, u_1, u_2, p. $\surd$

2. Solve the adjoint equations for ψ. $\surd$

3. Evaluate $\mathcal{G}$. $\surd$

4. Project $\mathcal{G}$ into an allowable subspace.

5. Update the shape.

6. Return to 1 until convergence is reached.

Practical implementation of the design method relies heavily upon fast and accurate solvers for both the state (w) and co-state (ψ) systems.

SEARCH PROCEDURE - 1

A simple descent method in which small steps are taken in the negative gradient direction is used.

$$\delta \mathcal{F} = -\lambda \mathcal{G}$$

can be regarded as simulating the time dependent process

$$\frac{d\mathcal{F}}{dt} = -\mathcal{G}$$

where λ is the time step Δt. Let A be the Hessian matrix with

$$A_{ij} = \frac{\partial \mathcal{G}_i}{\partial \mathcal{F}_j} = \frac{\partial^2 I}{\partial \mathcal{F}_i \partial \mathcal{F}_j}.$$

SEARCH PROCEDURE - 2

Suppose that a locally minimum value of the cost function $I^* = I(\mathcal{F}^*)$ is attained when $\mathcal{F} = \mathcal{F}^*$. Then the gradient $\mathcal{G}^* = \mathcal{G}(\mathcal{F}^*)$ must be zero, while the Hessian matrix $A^* = A(\mathcal{F}^*)$ must be positive definite. Since $\mathcal{G}^*$ is zero, the cost function can be expanded as a Taylor series in the neighborhood of $\mathcal{F}^*$ with the form

$$I(\mathcal{F}) = I^* + \frac{1}{2}\left(\mathcal{F} - \mathcal{F}^*\right) A \left(\mathcal{F} - \mathcal{F}^*\right) + \ldots$$

Correspondingly,

$$\mathcal{G}(\mathcal{F}) = A\left(\mathcal{F} - \mathcal{F}^*\right) + \ldots$$

SEARCH PROCEDURE - 3

As $\mathcal{F}$ approaches $\mathcal{F}^*$, the leading terms become dominant. Then, setting $\hat{\mathcal{F}} = (\mathcal{F} - \mathcal{F}^*)$, the search process approximates

$$\frac{d\hat{\mathcal{F}}}{dt} = -A^*\hat{\mathcal{F}}.$$

Also, since A^* is positive definite it can be expanded as

$$A^* = RMR^T,$$

where M is a diagonal matrix containing the eigenvalues of A^*, and

$$RR^T = R^T R = I.$$

SEARCH PROCEDURE - 4

Setting $$v = R^T \hat{\mathcal{F}},$$

the search process can be represented as $\frac{dv}{dt} = -Mv.$

The stability region for the simple forward Euler stepping scheme is a unit circle centered at -1 on the negative real axis. Thus for stability we must choose

$$\mu_{\mathsf{max}} \Delta t = \mu_{\mathsf{max}} \lambda < 2,$$

while the asymptotic decay rate, given by the smallest eigenvalue, is proportional to $e^{-\mu_{\mathsf{min}} t}$.

SEARCH PROCEDURE - 5

In order to improve the rate of convergence, one can set

$$\delta \mathcal{F} = -\lambda P \mathcal{G},$$

where P is a preconditioner for the search. An ideal choice is $P = A^{*-1}$, so that the corresponding time dependent process reduces to

$$\frac{d\hat{\mathcal{F}}}{dt} = -\hat{\mathcal{F}},$$

for which all the eigenvalues are equal to unity, and $\hat{\mathcal{F}}$ is reduced to zero in one time step by the choice $\Delta t = 1$.

QUASI-NEWTON METHODS

- Estimate A^* from the change in $\mathcal{G}$ during the search process.

- Requires accurate estimates of $\mathcal{G}$ at each time step.
  - Both the flow solution and adjoint equation must be fully converged.

- Most quasi-Newton methods also require a line search in each search direction, for which the flow equations and cost function must be accurately evaluated several times.

- They have proven quite robust for aerodynamic optimization (Reuther, Jameson, et al).

SMOOTHING THE GRADIENT

An alternative approach which has also proved extremely effective is to smooth the gradient by an implicit procedure, and to replace $\mathcal{G}$ by its smoothed value $\bar{\mathcal{G}}$ in the descent process. This both acts as a preconditioner, and ensures that each new shape in the optimization sequence remains smooth.

Interpretation $\Longrightarrow$

SOBOLEV GRADIENT EQUIVALENT - 1

Define a weighted Sobolev inner product

$$\langle u, v \rangle = \int_{\Omega} (uv + \epsilon \nabla u \cdot \nabla v) d\Omega \ ,$$

then

$$\langle u, v \rangle = (u, v) + (\epsilon \nabla u, \nabla v)$$

where the (u, v) is the standard inner product in L_2. Integration by parts yields

$$\langle u, v \rangle = \left(u - \nabla \left(\epsilon \nabla u\right), v\right) + \int_{\partial\Omega} \epsilon v \frac{\partial u}{\partial n} d\partial\Omega.$$

SOBOLEV GRADIENT EQUIVALENT - 2

Using the inner product notation the variation of the cost function I can be expressed as

$$\delta I = (\mathcal{G}, \delta\mathcal{F}) = \langle \bar{\mathcal{G}}, \delta\mathcal{F} \rangle = \left(\bar{\mathcal{G}} - \nabla \left(\epsilon \nabla \bar{\mathcal{G}} \right), \delta\mathcal{F} \right) \; .$$

Therefore we can solve implicitly for $\bar{\mathcal{G}}$

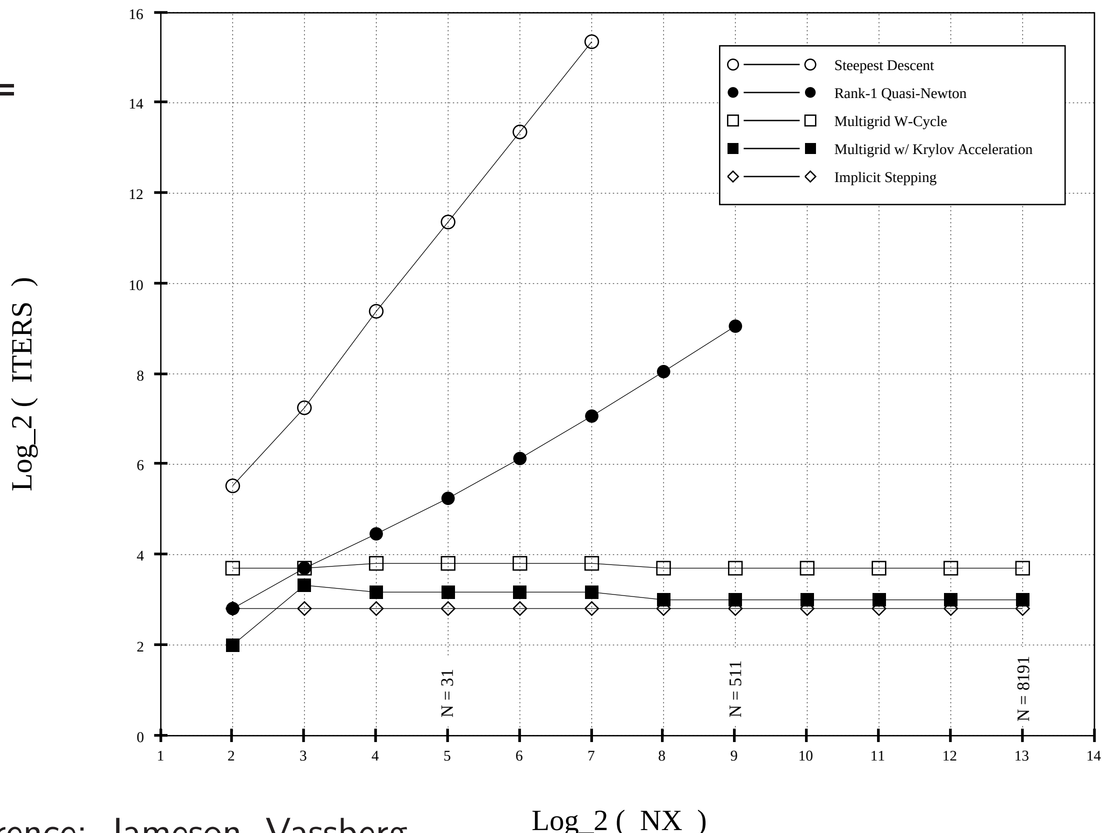
$$\bar{\mathcal{G}} - \nabla \left(\epsilon \nabla \bar{\mathcal{G}} \right) = \mathcal{G}.$$

Then can set $\delta\mathcal{F} = -\lambda \bar{\mathcal{G}}$,

$$\delta I = -\lambda \langle \bar{\mathcal{G}}, \bar{\mathcal{G}} \rangle = -\lambda \left(\mathcal{G}, \bar{\mathcal{G}} \right) \; .$$

COST OF SEARCH ALGORITHM

Steepest Descent	$\mathcal{O}(N^2)$ steps
Quasi-Newton	$\mathcal{O}(N)$ steps
Smoothed Gradient	$\mathcal{O}(K)$ steps



Reference: Jameson, Vassberg

COMPUTATIONAL COSTS

Finite Difference Gradients + Steepest Descent	$\mathcal{O}(N^3)$
Finite Difference Gradients + Quasi-Newton Search	$\mathcal{O}(N^2)$
Adjoint Gradients + Quasi-Newton Search	$\mathcal{O}(N)$
Adjoint Gradients + Smoothed Gradient Search (Note: K is independent of N)	$\mathcal{O}(K)$

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CURRENT STATUS - STRUCTURED MESH

- 3D RANS Design
  - Multiblock: Complex Configurations
  - Multiple Design Points
  - Successfully utilized in a wide variety of applications
    - Complete Transonic Aircraft Configurations
    - Rotating Machinery

CURRENT STATUS - UNSTRUCTURED MESH

- 2D Euler Design
- 3D Euler Design
  - Flow Solver: in place
  - Adjoint Solver: in place
  - Gradient Formulation: in place

FUTURE WORK

- General Geometry Definition and Modification
- Mesh Modification
- Parallel Implementation
- Extension to RANS